# LATENT GOLD® CHOICE 4.0

# USER'S GUIDE

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Statistical Innovations

*Thinking outside the brackets!™*

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# Manual for Latent GOLD Choice 4.0

## Structure of this manual

This manual consists of 3 Parts. Part 1 gives the overall general introduction to the program and new features.

Part 2, entitled the Technical Guide, documents all model options, technical features and output sections. It consists of 4 sections followed by a list of technical references. Section 1 presents a general introduction to the program. Section 2 contains several subsections which describe the various components of the model in formal mathematical terms, and provide examples of the various coding for the attributes. The last subsection (2.10) shows how all these components fit together in terms of the general latent class choice model. Section 3 describes the estimation, handling of missing data and other technical features. Section 4 provides the technical details for all of the output produced by the program.

Part 3 of the manual is entitled "Using Latent GOLD Choice". It lists all menus and contains a detailed tutorial which takes you through the use of the program with actual applications. The tutorial also illustrates the use of the different data formats.

In addition to this manual, users may wish to refer to the Latent GOLD 4.0 User's Guide, many of the details about the basic operation also apply to this program. That is, in addition to applying to the Cluster, DFactor and Regression modules of Latent GOLD 4.0 program, they also apply provide more complete operation details that also apply to Latent GOLD Choice 4.0

## Part 1: Overview

Latent GOLD Choice is available as a stand-alone program or as an optional add-on module for Latent GOLD 4.0. An optional Advanced Module is also available as well as an optional link to the SI-CHAID profiling package. Latent GOLD Choice supports the following kinds of latent class choice models:

- First choice models – An extended multinomial logit model (MNL) is used to estimate the probability of making a specific choice among a set of alternatives as a function of choice *attributes* and individual characteristics (*predictors*).
- Ranking models – The sequential logit model is used for situations where a $1^{st}$ and $2^{nd}$ choice, $1^{st}$ and last choice (best-worst), other partial rankings or choices from a complete ranking of all alternatives are obtained.
- Conjoint rating models – An ordinal logit model is used for situations where ratings of various alternatives, which may be viewed as a special kind of choice, are obtained.

For each of these situations, response data are obtained for one or more replications known as choice sets. Latent class (LC) choice models account for heterogeneity in the data by allowing for the fact that different population segments (latent classes) express different preferences in making their choices. For any application, separate models may be estimated that specify different numbers of classes. Various model fit statistics and other output are provided to compare these models to assist in determining the actual number of classes. *Covariates* may also be included in the model for improved description/ prediction of the segments.

Advanced program features include the use of various weights – case weights, replication weights, scale weights, and options to restrict the part-worth utility estimates in various ways. Some of the advanced applications include:

- Allocation models – Replication weights may be used to handle designs where respondents allocate a number of votes (purchases, points) among the various choice alternatives.
- Best-worst and related models -- A scale factor of '–1' can be used to specify the alternative(s) judged to be *worst* (or least preferred) as opposed to *best* (or most preferred).

## Latent GOLD Choice 4.0 Advanced

The following new features are included in the optional Advanced Module (requires the Advanced version) of Latent GOLD Choice 4.0:

- **Continuous latent variables** (CFactors) – an option for specifying models containing continuous latent variables, called CFactors, in a choice, ranking or rating model. CFactors can be used to specify random-coefficients conditional logit models, in which the random coefficients covariance matrix is restricted using a factor-analytic structure. It is also possible to use random-effects in conjunction with latent classes, yielding hybrid choice models combining discrete and continuous unobserved heterogeneity. If included, additional information pertaining to the CFactor effects appear in the Parameters, ProbMeans, and the Classification Statistics output and CFactor scores appear in the Standard Classification output.
- **Multilevel modeling** – an option for specifying LC choice models for nested data structures such as shoppers (individuals) within stores (groups). Group-level variation may be accounted for by specifying *group-level latent classes* (GClasses) and/or group-level CFactors (GCFactors). In addition, when 2 or more GClasses are specified, group-level covariates (GCovariates) can be included in the model to describe/predict them.
- **Survey options** for dealing with complex sampling data. Two important survey sampling designs are stratified sampling -- sampling cases within strata, and two-stage cluster sampling -- sampling within primary sampling units (PSUs) and subsequent sampling of cases within the selected PSUs. Moreover, sampling weights may exist. The Survey option takes the sampling design and the sampling weights into account when computing standard errors and related statistics associated with the parameter estimates, and estimates the 'design. The parameter estimates are the same as when using the weight variable as a Case Weight when this method is used. An alternative two-step approach ('unweighted') proposed in Vermunt and Magidson (2001) is also available for situations where the weights may be somewhat unstable.

## Additional Optional Add-ons to Latent GOLD Choice 4.0

The following optional add-on programs are also available that link to Latent GOLD Choice 4.0:

### Latent GOLD 4.0

A license to Latent GOLD 4.0 allows you to use a single fully integrated latent class program that contains the Choice program as one module, and includes 3 additional modules to allow estimation of LC Cluster, DFactor (Discrete Factors) , and LC Regression models.

Numerous tutorials, and articles illustrate the use of these 3 kinds of models at

http://www.statisticalinnovations.com/products/latentgold_v4.html

In addition, the complete LG 4.0 User's Guide and a separate Technical Guide may also be downloaded.

### SI-CHAID® 4.0

With this option, a CHAID (CHi-squared Automatic Interaction Detector) analysis may be performed following the estimation of any LC Choice model, to profile the resulting LC segments based on demographics and/or other exogenous variables (*Covariates*). By selecting 'CHAID' as one of the output options, a CHAID input file is constructed upon completion of the model estimation, which can then be used as input to SI-CHAID 4.0.

This option provides an alternative treatment to the use of active and/or inactive covariates in Latent GOLD Choice 4.0. In addition to standard Latent GOLD output to examine the relationship between the covariates and classes/DFactors, SI-CHAID provides a tree-structured profile of selected classes/DFactors based on the selected Covariates. In addition, chi-square measures of statistical significance are provided for *all* covariates (Latent GOLD Choice does not provide such for *inactive* covariates).

Whenever covariates are available to describe latent classes obtained from Latent GOLD Choice 4.0, SI-CHAID 4.0 can be an especially valuable add-on tool under any of the following conditions:

- when *many* covariates are available and you wish to know which ones are most important
- when you do not wish to specify certain covariates as active because you do not wish them to affect the model parameters, but you still desire to assess their statistical significance with respect to the classes (or a specified subset of the classes)
- when you wish to develop a separate profile for each latent class (see Tutorial #1A)
- when you wish to explore differences between 2 or more selected latent classes using a tree modeling structure
- when the relationship between the covariates and classes is nonlinear or includes interaction effects, or
- when you wish to profile order-restricted latent classes
- For an example of the use of CHAID with Latent GOLD Choice 4.0, see Latent GOLD Choice Tutorial #1A on our website.

This option is especially useful in the development of simulators, as simulators can be easily extended to predict shares not only for each latent class segment, but also for CHAID segments defined using relevant exogeneous variables.

For further information on the CHAID add-on option, see:

http://www.statisticalinnovations.com/products/chaid_v4.html

## DBMS/Copy interface

Latent GOLD Choice 4.0 reads SPSS, .cho and ASCII text files for data input. The DBMS/Copy interface allows Latent GOLD Choice 4.0 to directly open over 80 additional file formats, including Excel, SAS and HTML files. The full list of file formats is available at http://www.statisticalinnovations.com/products/latentgold_80formats.html


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# Technical Guide

# for Latent GOLD Choice 4.0:

# Basic and Advanced[1]

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# Part I: Basic Model Options, Technical Settings, and Output Sections

## 1 Introduction to Part I (Basic Models)

Latent GOLD Choice is a program for the analysis of various types of preference data; that is, data containing (partial) information on respondents' preferences concerning one or more sets of alternatives, objects, options, or products. Such data can be obtained from different response formats, the most important of which are:

- first choice out of a set of $M$ alternatives, possibly including a none option,
- paired comparison,
- full ranking of a set of $M$ alternatives,
- partial ranking of a set of $M$ alternatives,
- best and worst choices out of a set of $M$ alternatives; also referred to as maximum-difference scaling,
- binary rating of a single alternative (positive-negative, yes-no, like-dislike),
- polytomous rating of a single alternative, e.g., on a 5-point scale,
- assigning a probability to a set of $M$ alternatives; also referred to as constant-sum data,
- distribution of a fixed number of points (votes, chips, dollars, etc.) among a set of $M$ alternatives; also referred to as an allocation format,
- pick any out of a set of $M$ alternatives,
- pick $k$ (a prespecified number) out of a set of $M$ alternatives, which is an example of what can be called a joint choice.

Latent GOLD Choice will accept each of these formats, including any combination of these.

The purpose of a discrete choice analysis is to predict stated or revealed preferences from characteristics of alternatives, choice situations, and respondents. The regression model that is used for this purpose is the conditional logit model developed by McFadden (1974). This is an extended multinomial logit model that allows the inclusion of characteristics of the alternatives – attributes such as price – as explanatory variables. Although the conditional logit model was originally developed for analyzing *first* choices, each of the other response formats can also be handled by adapting the basic model to the format concerned. For example, a ranking task is treated as a sequence of first choices, where the alternatives selected previously are eliminated; a rating task is modelled by an adjacent-category ordinal logit model, which is a special type of conditional logit model for ordinal outcome variables.

Latent GOLD Choice is not only a program for modeling choices or preferences, but also a program for latent class (LC) analysis. A latent class or finite mixture structure is used to capture preference heterogeneity in the population of interested. More precisely, each latent class corresponds to a population segment that differs with respect to the importance (or weight) given to the attributes of the alternatives when expressing that segment's preferences. Such a discrete characterization of unobserved heterogeneity is sometimes referred to as a nonparametric random-coefficients approach (Aitkin, 1999; Laird, 1978; Vermunt, 1997; Vermunt and Van Dijk, 2001; Vermunt and Magidson, 2003). Latent GOLD Choice implements a nonparametric variant of the random-coefficient or mixed conditional logit model (Andrews et al., 2002; Louviere et al., 2000; McFadden and Train, 2000). The LC choice model can also be seen as a variant of the LC or mixture regression model (Vermunt and Magidson, 2000; Wedel and DeSarbo, 1994, 2002).

Most studies will contain multiple observations or multiple replications per respondent: e.g., respondents indicate their first choice for several sets of products or provide ratings for various products. This introduces dependence between observations. It is this dependence caused by the repeated measures that makes it possible to obtain stable estimates of the class-specific regression parameters.

A third aspect of the model implemented in Latent GOLD Choice is that class membership can be predicted from individual characteristics (covariates). In other words, one can not only identify latent classes, clusters, or segments that differ with respect to their preferences, but it is also possi-

ble to predict to which (unobserved) subgroup an individual belongs using covariates. Such a profiling of the latent classes substantially increases the practical usefulness of the results and improves out-of-study prediction of choices (Magidson, Eagle, and Vermunt, 2003; Natter and Feurstein, 2002; Vermunt and Magidson, 2002).

The next section describes the LC models implemented in Latent GOLD Choice. Then, attention is paid to estimation procedures and the corresponding technical options of the program. The output provided by the program is described in the last section.

Several tutorials are available to get you up and running quickly. These include:

- cbcRESP.sav - a simulated choice experiment
  - Tutorial 1: Using LG Choice to Estimate Discrete Choice Models
  - Tutorial 2: Using LG Choice to Predict Future Choices
- brandABresp.sav - a simulated brand-price choice experiment
  - Tutorial 3: Estimating Brand and Price Effects
  - Tutorial 4: Using the 1-file Format
- bank45.sav & bank9-1-file.sav - real data from a bank segmentation study
  - Tutorial 5: Analyzing Ranking Data
  - Tutorial 6: Using LG Choice to Estimate max-diff (best-worst) and Other Partial Ranking Models
- conjoint.sav, ratingRSP.sav, ratingALT.sav, ratingSET.sav: simulated data utilizing a 5-point ratings scale
  - Tutorial 7: LC Segmentation with Ratings-based Conjoint Data
  - Tutorial 7A: LC Segmentation with Ratings-based Conjoint Data

All of the above tutorials are available on our website at
http://www.statisticalinnovations.com/products/choice.html#tutorialslink

# 2 The Latent Class Model for Choice Data

In order to be able to describe the models of interest, we first must clarify some concepts and introduce some notation. The data file contains information on $I$ cases or subjects, where a particular case is denoted by $i$. For each case, there are $T_i$ replications, and a particular replication is denoted by $t$. Note that in the Latent GOLD Choice interface, the $T_i$ observations belonging to the same case are linked by means of a common *Case ID*.

Let $y_{it}$ denote the value of the dependent (or response) variable for case $i$ at replication $t$, which can take on values $1 \leq m \leq M$. In other words, $M$ is number of alternatives and $m$ a particular alternative. Three types of explanatory variables can be used in a LC choice model: attributes or characteristics of alternatives ($z^{att}_{itmp}$), predictors or characteristics of replications ($z^{pre}_{itq}$), and covariates or characteristics of individuals ($z^{cov}_{ir}$). Here, the indices $p$, $q$, and $r$ are used to refer to a particular attribute, predictor, and covariate. The total number of attributes, predictors, and covariates is denoted by $P$, $Q$, and $R$, respectively. Below, we will sometimes use vector notation $\mathbf{y}_i$, $\mathbf{z}_i$, and $\mathbf{z}^{cov}_i$ to refer to all responses, all explanatory variables, and all covariate values of case $i$, and $\mathbf{z}^{att}_{it}$ and $\mathbf{z}^{pre}_{it}$ to refer to the attribute and predictor values corresponding to replication $t$ for case $i$. Another variable that plays an important role in the models discussed below is the latent class variable denoted by $x$, which can take on values $1 \leq x \leq K$. In other words, the total number of latent classes is denoted by $K$.

Two other variables that may be used in the model specification are a replication-specific scale factor $s_{it}$ and a replication-specific weight $v_{it}$. Their default values are one, in which case they do not affect the model structure.

## 2.1 First Choices

We start with the description of the regression model for the simplest and most general response format, first choice. For simplicity of exposition, we assume that each replication or choice set has the same number of alternatives. Later on it will be shown how to generalize the model to other formats, including choice sets with unequal numbers of alternatives.

A conditional logit model is a regression model for the probability that case $i$ selects alternative $m$ at replication $t$ given attribute values $\mathbf{z}^{att}_{it}$ and predictor values $\mathbf{z}^{pre}_{it}$. This probability is denoted by $P(y_{it} = m|\mathbf{z}^{att}_{it}, \mathbf{z}^{pre}_{it})$. Attributes are characteristics of the alternatives; that is, alternative $m$ will

have different attribute values than alternative $m'$. Predictors on the other hand, are characteristics of the replication or the person, and take on the same value across alternatives. For the moment, we assume that attributes and predictors are numeric variables. In the subsection " Coding of Nominal Variables", we explain in detail how nominal explanatory variables are dealt with by the program.

The conditional logit model for the response probabilities has the form

$$P(y_{it} = m|\mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre}) = \frac{\exp(\eta_{m|\mathbf{z}_{it}})}{\sum_{m'=1}^{M} \exp(\eta_{m'|\mathbf{z}_{it}})},$$

where $\eta_{m|\mathbf{z}_{it}}$ is the systematic component in the utility of alternative $m$ for case $i$ at replication $t$. The term $\eta_{m|\mathbf{z}_{it}}$ is a linear function of an alternative-specific constant $\beta_m^{con}$, attribute effects $\beta_p^{att}$, and predictor effects $\beta_{mq}^{pre}$ (McFadden, 1974). That is,

$$\eta_{m|\mathbf{z}_{it}} = \beta_m^{con} + \sum_{p=1}^{P} \beta_p^{att}\, z_{itmp}^{att} + \sum_{q=1}^{Q} \beta_{mq}^{pre}\, z_{itq}^{pre},$$

where for identification purposes $\sum_{m=1}^{M} \beta_m^{con} = 0$, and $\sum_{m=1}^{M} \beta_{mq}^{pre} = 0$ for $1 \leq q \leq Q$, a restriction that is known as effect coding. It is also possible to use dummy coding using either the first or last category as reference category (see subsection 2.8). Note that the regression parameters corresponding to the predictor effects contain a subscript $m$, indicating that their values are alternative specific.

The inclusion of the alternative-specific constant $\beta_m^{con}$ is optional, and models will often not include predictor effects. Without alternative-specific constants and without predictors, the linear model for $\eta_{m|\mathbf{z}_{it}}$ simplifies to

$$\eta_{m|\mathbf{z}_{it}} = \sum_{p=1}^{P} \beta_p^{att}\, z_{itmp}^{att}.$$

In a latent class or finite mixture variant of the conditional model, it is assumed that individuals belong to different latent classes that differ with respect to (some of) the $\beta$ parameters appearing in the linear model for $\eta$ (Kamakura and Russell, 1989). In order to indicate that the choice probabilities depend on class membership $x$, the logistic model is now of the form

$$P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre}) = \frac{\exp(\eta_{m|x,\mathbf{z}_{it}})}{\sum_{m'=1}^{M} \exp(\eta_{m'|x,\mathbf{z}_{it}})}. \quad (1)$$

Here, $\eta_{m|x,\mathbf{z}_{it}}$ is the systematic component in the utility of alternative $m$ at replication $t$ given that case $i$ belongs to latent class $x$. The linear model for $\eta_{m|x,\mathbf{z}_{it}}$ is

$$\eta_{m|x,\mathbf{z}_{it}} = \beta_{xm}^{con} + \sum_{p=1}^{P} \beta_{xp}^{att} \, z_{itmp}^{att} + \sum_{q=1}^{Q} \beta_{xmq}^{pre} \, z_{itq}^{pre}. \tag{2}$$

As can be seen, the only difference with the aggregate model is that the logit regression coefficients are allowed to be Class specific.

In the LC choice model, the probability density associated with the responses of case $i$ has the form

$$P(\mathbf{y}_i|\mathbf{z}_i) = \sum_{x=1}^{K} P(x) \prod_{t=1}^{T_i} P(y_{it}|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred}). \tag{3}$$

Here, $P(x)$ is the unconditional probability of belonging to Class $x$ or, equivalently, the size of latent class $x$. Below, it will be shown that this probability can be allowed to depend an individual's covariate values $\mathbf{z}_i^{cov}$, in which case $P(x)$ is replaced by $P(x|\mathbf{z}_i^{cov})$.

As can be seen from the probability structure described in equation (3), the $T_i$ repeated choices of case $i$ are assumed to be independent of each other given class membership. This is equivalent to the assumption of local independence that is common in latent variable models, including in the traditional latent class model (Bartholomew and Knott, 1999; Goodman, 1974a, 1974b; Magidson and Vermunt, 2004). Also in random-coefficients models, it is common to assume responses to be independent conditional on the value of the random coefficients.

## 2.2 Rankings and Other Situations with Impossible Alternatives

In addition to models for (repeated) first choices, it is possible to specify models for rankings. One difference between first-choice and ranking data is that in the former there is a one-to-one correspondence between replications and choice sets while this is no longer the case with ranking data. In a ranking task, the number of replications generated by a choice sets equals the number of choices that is provided. A full ranking of a choice set consisting of five alternatives yields four replications; that is, the first, second, third, and fourth choice. Thus, a set consisting of $M$ alternatives, generates $M-1$

replications. This is also the manner in which the information appears in the response data file. With partial rankings, such as first and second choice, the number of replications per set will be smaller.

The LC model for ranking data implemented in Latent GOLD Choice treats the ranking task as sequential choice process (Böckenholt, 2002; Croon, 1989; Kamakura et al., 1994). More precisely, each subsequent choice is treated as if it were a first choice out of a set from which alternatives that were already selected are eliminated. For example, if a person's first choice out of a set of 5 alternatives is alternative 2, the second choice is equivalent to a (first) choice from the 4 remaining alternatives 1, 3, 4, and 5. Say that the second choice is 4. The third choice will then be equivalent to a (first) choice from alternatives 1, 3, and 5.

The only adaptation that is needed for rankings is that it should be possible to have a different number of alternatives per set or, in our terminology, that certain alternatives are "impossible". More precisely, $M$ is still the (maximum) number of alternatives, but certain alternatives cannot be selected in some replications. In order to express this, we need to generalize our notation slightly. Let $A_{it}$ denote the set of "possible" alternatives at replication $t$ for case $i$. Thus, if $m \in A_{it}$, $P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre})$ is a function of the unknown regression coefficients, and if $m \notin A_{it}$, $P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre}) = 0$. An easy way to accomplish this without changing the model structure is by setting $\eta_{m|x,\mathbf{z}_{it}} = -\infty$ for $m \notin A_{it}$. Since $\exp(-\infty) = 0$, the choice probability appearing in equation (1) becomes:

$$P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre}) = \frac{\exp(\eta_{m|x,\mathbf{z}_{it}})}{\sum_{m' \in A_{it}} \exp(\eta_{m'|x,\mathbf{z}_{it}})}$$

if $m \in A_{it}$ and $P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre}) = 0$ if $m \notin A_{it}$. As can be seen, the sum in the denominator is over the possible alternatives only.

When the dependent variable is specified to be a ranking variable, the specification of those alternatives previously selected as impossible alternatives is handled automatically by the program. The user can use a missing value in the sets file to specify alternatives as impossible. This makes it possible to analyze choice sets with different numbers of alternatives per set, as well as combinations of different choice formats. In the one-file data format, choice sets need not have the same numbers of alternatives. In this case, the program treats "unused" alternatives as "impossible" alternatives.

## 2.3 Ratings

A third type of dependent variable that can be dealt with are preferences in the form of ratings. Contrary to a first choice or ranking task, a rating task concerns the evaluation of a single alternative instead of the comparison of a set of alternatives. Attributes will, therefore, have the same value across the categories of the response variable. Thus for rating data, it is no longer necessary to make a distinction between attributes and predictors.

Another important difference with first choices and rankings is that ratings outcome variables should be treated as ordinal instead of nominal. For this reason, we use an adjacent-category ordinal logit model as the regression model for ratings (Agresti, 2002; Goodman, 1979; Magidson, 1996). This is a restricted multinomial/conditional logit model in which the category scores for the dependent variable play an important role. Let $y_m^*$ be the score for category $m$. In most cases, this will be equally-spaced scores with mutual distances of one – e.g., 1, 2, 3, ... $M$, or 0, 1, 2, ... $M-1$ – but it is also possible to use scores that are not equally spaced or non integers. Note that $M$ is no longer the number of alternatives in a set but the number of categories of the response variable.

Using the same notation as above, the adjacent-category ordinal logit model can be formulated as follows

$$\eta_{m|x,\mathbf{z}_{it}} = \beta_{xm}^{con} + y_m^* \cdot \left( \sum_{p=1}^{P} \beta_{xp}^{att} \, z_{itp}^{att} + \sum_{q=1}^{Q} \beta_{xq}^{pre} \, z_{itq}^{pre} \right).$$

The attribute and predictor effects are multiplied by the fixed category score $y_m^*$ to obtain the systematic part of the "utility" of rating $m$ . As can be seen, there is no longer a fundamental difference between attributes and predictors since attribute values and predictor effects no longer depend on $m$. For ratings, $\eta_{m|x,\mathbf{z}_{it}}$ is defined by substituting $y_m^* \cdot z_{itp}^{att}$ in place of the category-specific attribute values $z_{itmp}^{att}$ in equation (2), and the category-specific predictor effects $\beta_{xmq}^{pre}$ are replaced by $y_m^8 \cdot \beta_{xq}^{pre}$. The relationship between the category-specific utilities $\eta_{m|x,\mathbf{z}_{it}}$ and the response probabilities is the same as in the model for first choices (see equation 1).

As mentioned above, in most situations the category scores $y_m^*$ are equally spaced with a mutual distance of one. In such cases, $y_m^* - y_{m-1}^* = 1$, and as result

$$\log \frac{P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre})}{P(y_{it} = m-1|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre})} \quad = \quad \eta_{m|x,\mathbf{z}_{it}} - \eta_{m-1|x,\mathbf{z}_{it}}$$

$$= \left(\beta_{xm}^{con} - \beta_{x(m-1)}^{con}\right) + \sum_{p=1}^{P} \beta_{xp}^{att}\, z_{itp}^{att} + \sum_{q=1}^{Q} \beta_{xq}^{pre}\, z_{itq}^{pre}.$$

This equation clearly shows the underlying idea behind the adjacent-category logit model. The logit in favor of rating $m$ instead of $m-1$ has the form of a standard binary logit model, with an intercept equal to $\beta_{xm}^{con} - \beta_{x(m-1)}^{con}$ and slopes equal to $\beta_{xq}^{att}$ and $\beta_{xq}^{pre}$. The constraint implied by the adjacent-category ordinal logit model is that the slopes are the same for each pair of adjacent categories. In other words, the attribute and predictor effects are the same for the choice between ratings 2 and 1 and the choice between ratings 5 and 4.

## 2.4 Replication Scale and Best-Worst Choices

A component of the LC choice model implemented in Latent GOLD Choice that has not been introduced thus far is the replication-specific scale factor $s_{it}$. The scale factor allows the utilities to be scaled differently for certain replications. Specifically, the scale factor enters into the conditional logit model in the following manner:

$$P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre}, s_{it}) = \frac{\exp(s_{it} \cdot \eta_{m|x,\mathbf{z}_{it}})}{\sum_{m'=1}^{M} \exp(s_{it} \cdot \eta_{m'|x,\mathbf{z}_{it}})}.$$

Thus, it is seen that while the scale factor is assumed to be constant across alternatives within a replication, it can take on different values between replications. The form of the linear model for $\eta_{m|x,\mathbf{z}_{it}}$ is not influenced by the scale factors and remains as described in equation (2). Thus, the scale factor allows for a different scaling of the utilities across replications. The default setting for the scale factor is $s_{it} = 1$, in which case it cancels from the model for the choice probabilities.

Two applications of this type of scale factor are of particular importance in LC Choice modeling. The first is in the analysis of best-worst choices or maximum-difference scales (Cohen, 2003). Similar to a partial ranking task, the selection of the best and worst alternatives can be treated as a sequential choice process. The selection of the best option is equivalent to a first choice. The selection of the worst alternative is a (first) choice out of the remaining alternatives, where the choice probabilities are negatively related

to the utilities of these alternatives. By declaring the dependent variable to be a ranking, the program automatically eliminates the best alternative from the set available for the second choice. The fact that the second choice is not the second best but the worst can be indicated by means of a replication scale factor of -1, which will reverse the choice probabilities. More precisely, for the worst choice,

$$P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre}, s_{it}) = \frac{\exp(-1 \cdot \eta_{m|x,\mathbf{z}_{it}})}{\sum_{m' \in A_{it}} \exp(-1 \cdot \eta_{m'|x,\mathbf{z}_{it}})}$$

if $m \in A_{it}$ and 0 if $m \notin A_{it}$.

The second noteworthy application of the scale factor occurs in the simultaneous analysis of stated and revealed preferences. Note that use of a scale factor larger than 0 but smaller than 1 causes $s_{it} \cdot \eta_{m|x,\mathbf{z}_{it}}$ to be shrunk compared to $\eta_{m|x,\mathbf{z}_{it}}$ and as a result, the choice probabilities become more similar across alternatives. A well-documented phenomenon is that stated preferences collected via questionnaires yield more extreme choice probabilities than revealed preferences (actual choices) even if these utilities are the same (Louviere et al., 2000). A method to transform the utilities for these two data types to the same scale is to use a somewhat smaller scale factor for the revealed preferences than for the stated preferences. Assuming that the scale factor for the stated preferences is 1.0, values between 0.5 and 1.0 could be tried out for the revealed preferences; for example,

$$P(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pre}, s_{it}) = \frac{\exp(0.75 \cdot \eta_{m|x,\mathbf{z}_{it}})}{\sum_{m'=1}^{M} \exp(0.75 \cdot \eta_{m'|x,\mathbf{z}_{it}})}.$$

A limitation of the scale factor implemented in Latent GOLD Choice is that it cannot vary across alternatives. However, a scale factor is nothing more than a number by which the attributes (and predictors) are multiplied, which is something that users can also do themselves when preparing the data files for the analysis. More precisely, the numeric attributes of the alternatives may be multiplied by the desired scale factor.

## 2.5 Replication Weight and Constant-sum Data

The replication weights $v_{it}$ modify the probability structure defined in equation (3) as follows:

$$P(\mathbf{y}_i|\mathbf{z}_i) = \sum_{x=1}^{K} P(x) \prod_{t=1}^{T_i} \left[P(y_{it}|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred})\right]^{v_{it}}.$$

The interpretation of a weight is that choice $y_{it}$ is made $v_{it}$ times.

One of the applications of the replication weight is in the analysis of constant-sum or allocation data. Instead of choosing a single alternative out of set, the choice task may be to attach a probability to each of the alternatives. These probabilities serve as replication weights. Note that with such a response format, the number of replications corresponding to a choice set will be equal to the number of alternatives. A similar task is the distribution of say 100 chips or coins among the alternatives, or a task with the instruction to indicate how many out of 10 visits of a store one would purchase each of several products presented.

Other applications of the replication weights include grouping and differential weighting of choices. Grouping may be relevant if the same choice sets are offered several times to each observational unit. Differential weighting may be desirable when analyzing ranking data. In this case, the first choice may be given a larger weight in the estimation of the utilities than subsequent choices. It is even possible to ask respondents to provide weights – say between 0 and 1 – to indicate how certain they are about their choices. In the simultaneous analysis of stated and revealed preference data, it is quite common that several stated preferences are combined with a single revealed preference. In such a case, one may consider assigning a higher weight to the single revealed preference replication to make sure that both preference types have similar effects on the parameter estimates.

## 2.6 Other Choice/Preference Formats

In the previous sections, we showed how to deal with most of the response formats mentioned in the introduction. To summarize, first choice is the basic format, rankings are dealt with as sequences of first choices with impossible alternatives, ratings are modelled by an ordinal logit model, best-worst choices can be treated as partial rankings with negative scale factors for the second (worst) choice, and the analysis of constant-sum data involves the use of replications weights.

A format that has not been discussed explicitly is paired comparisons (Dillon and Kumar, 1994). Paired comparisons are, however, just first choices out of sets consisting of two alternatives, and can therefore be analyzed in the same way as first choices. Another format mentioned in the introduction is binary rating. Such a binary outcome variable concerning the evaluation of a single alternative (yes/no, like/dislike) can simply be treat as a rating. The

most natural scoring of the categories would be to use score 1 for the positive response and 0 for the negative response, which yields a standard binary logit model. The pick any out of $M$ format can be treated in the same manner as binary ratings; that is, as a set of binary variables indicating whether the various alternatives are picked or not.

Another format, called joint choices, occurs if a combination of two or more outcome variables are modelled jointly. Suppose the task is to give the two best alternatives out of a set of $M$, which is a pick 2 out of $M$ format. This can be seen as a single choice with $M \cdot (M-1)/2$ joint alternatives. The attribute values of these joint alternatives are obtained by summing the attribute values of the original pair of alternatives. Other examples of joint choices are non-sequential models for rankings (Böckenholt, 2002; Croon, 1989) and best-worst choices (Cohen, 2003). For example, a pair of best and worst choices can also be seen as a joint choice out of $M \cdot (M-1)$ joint alternatives. The attribute values of these joint alternatives are equal to the attribute values of the best minus the attributes of the worst. What is clear from these examples is that setting up a model for a joint choice can be quite complicated.

Another example of a situation in which one has to set up a model for a joint response variable is in capture-recapture studies (Agresti, 2002). For the subjects that are captured at least ones, one has information on capture at the various occasions. The total number of categories of the joint dependent variable is $2^T - 1$, where $T$ is the number of time point or replications.

Note that these examples of joint choice models all share the fact that the number of possible joint alternatives is smaller than the product of the number of alternatives of the separate choices. That is, in each case, certain combinations of choices are impossible, and hence the model of interest cannot be set up as a series of independent choices. Instead, these situations should be specified as a single joint choice.

The last "choice format" we would like to mention is the combination of different response formats. The most general model is the model for first choices. Models for rankings and ratings are special cases that are obtained by adapting the model for first choices to the response format concerned. This is done internally (automatically) by the program. It is, however, also possible to specify ranking or rating models as if they were models for first choices. In the ranking case, this involves specifying additional choice sets in which earlier selected alternatives are defined as "impossible". A rating model can be specified as a first choice model by defining the categories of

the dependent variable as alternatives with attribute values $z_{itpm}^{att}$ equal to $y_m^* \cdot z_{itp}^{att}$.

Given the fact that each of the response formats can be treated as a first choice, it is possible to make any combination of the formats that were discussed. Of course, setting up the right alternatives and sets files may be quite complicated. An issue that should be taken into account when using combinations of response formats is the scaling of the replications. For example, it might be that the utilities should be scaled in a different manner for ratings than for first choices.

## 2.7 Covariates

In addition to the explanatory variables that we called attributes and predictors, it is also possible to include another type of explanatory variable – called covariates – in the LC model. While attributes and predictors enter in the regression model for the choices, covariates are used to predict class membership. In the context of LC analysis, covariates are sometimes referred to as concomitant or external variables (Clogg, 1981; Dayton and McReady, 1988; Kamakura et al., 1994; Van der Heijden et al., 1996).

When covariates are included in the model, the probability structure changes slightly compared to equation (3). It becomes

$$P(\mathbf{y}_i|\mathbf{z}_i) = \sum_{x=1}^{K} P(x|\mathbf{z}_i^{cov}) \prod_{t=1}^{T_i} P(y_{it}|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred}). \tag{4}$$

As can be seen, class membership of individual $i$ is now assumed to depend on a set of covariates denoted by $\mathbf{z}_i^{cov}$. A multinomial logit is specified in which class membership is regressed on covariates; that is,

$$P(x|\mathbf{z}_i^{cov}) = \frac{\exp(\eta_{x|\mathbf{z}_i})}{\sum_{x'=1}^{K} \exp(\eta_{x'|\mathbf{z}_i})},$$

with linear term

$$\eta_{x|\mathbf{z}_i} = \gamma_{0x} + \sum_{r=1}^{R} \gamma_{rx} z_{ir}^{cov}. \tag{5}$$

Here, $\gamma_{0x}$ denotes the intercept or constant corresponding to latent class $x$ and $\gamma_{rx}$ is the effect of the $r$th covariate for Class $x$. Similarly to the model for the choices, for identification, we either set $\sum_{x=1}^{K} \gamma_{rx} = 0$, $\gamma_{r1} = 0$, or

$\gamma_{rK} = 0$ for $0 \leq r \leq R$, which amounts to using either effect or dummy coding. Although in equation (5) the covariates are assumed to be numeric, the program can also deal with nominal covariates (see subsection 2.8).

We call this procedure for including covariates in a model the "active covariates method": Covariates are active in the sense that the LC choice solution with covariates can be somewhat different from the solution without covariates. An alternative method, called "inactive covariates method", involves computing descriptive measures for the association between covariates and the latent variable after estimating a model without covariate effects. More detail on the latter method is given in the subsection explaining the Profile and ProbMeans output.

Another approach that can be used to explore the relationship between covariates and the latent variable is through the use of the CHAID option. This option may be especially valuable when the goal is to profile the latent classes using many inactive covariates. This option requires the SI-CHAID 4.0 add-on program, which assesses the statistical significance between each covariate and the latent variable. For further information about the CHAID option see Section 4.9.

## 2.8 Coding of Nominal Variables

In the description of the LC choice models of interest, we assumed that attributes, predictors, and covariates were all numeric. This limitation is not necessary however, as Latent GOLD Choice allows one or more of these explanatory variables to be specified to be nominal. For nominal variables, Latent GOLD Choice sets up the design vectors using either effect (ANOVA-type) coding or dummy coding with the first or last category as reference category for identification. Effect coding means that the parameters will sum to zero over the categories of the nominal variable concerned, In dummy coding, the parameters corresponding to the reference category are fixed to zero.

Suppose we have a nominal attribute with 4 categories in a model for first choices. The effect coding constraint implies that the corresponding 4 effects should sum to 0. This is accomplished by defining a design matrix with 3 numeric attributes $z^{att}_{it1m}$, $z^{att}_{it2m}$, and $z^{att}_{it3m}$. The design matrix that is set up

for the 3 non-redundant terms ($\beta_{x1}^{att}$, $\beta_{x2}^{att}$, $\beta_{x3}^{att}$) is as follows:

$$\begin{array}{lrrr} \text{category 1} & 1 & 0 & 0 \\ \text{category 2} & 0 & 1 & 0 \\ \text{category 3} & 0 & 0 & 1 \\ \text{category 4} & -1 & -1 & -1 \end{array} ,$$

where each row corresponds to a category of the attribute concerned and each column to one of the three parameters. Although the parameter for the last category is omitted from model, you do not notice that because it is computed by the program after the model is estimated. The parameter for the fourth category equals $-\sum_{p=1}^{3} \beta_{xp}^{att}$; that is, minus the sum of the parameters of the three other categories. This guarantees that the parameters sum to zero since $\sum_{p=1}^{3} \beta_{xp}^{att} - \sum_{p=1}^{3} \beta_{xp}^{att} = 0$.

Instead of using effect coding, it is also possible to use dummy coding. Depending on whether one uses the first or the last category as reference category, the design matrix will look like this

$$\begin{array}{lrrr} \text{category 1} & 0 & 0 & 0 \\ \text{category 2} & 1 & 0 & 0 \\ \text{category 3} & 0 & 1 & 0 \\ \text{category 4} & 0 & 0 & 1 \end{array} .$$

or this

$$\begin{array}{lrrr} \text{category 1} & 1 & 0 & 0 \\ \text{category 2} & 0 & 1 & 0 \\ \text{category 3} & 0 & 0 & 1 \\ \text{category 4} & 0 & 0 & 0 \end{array} .$$

Whereas in effect coding the category-specific effects should be interpreted in terms of deviation from the average, in dummy coding their interpretation is in terms of difference from the reference category. Note that the parameter for the reference category is omitted, which implies that it is equated to 0.

It also possible to work with user specified coding schemes. An example is

$$\begin{array}{lrrr} \text{category 1} & 0 & 0 & 0 \\ \text{category 2} & 1 & 0 & 0 \\ \text{category 3} & 1 & 1 & 0 \\ \text{category 4} & 1 & 1 & 1 \end{array} ,$$

which yields parameters that can be interpreted as differences between adjacent categories. More precisely, $\beta_{x1}^{att}$ is the difference between categories 2 and 1, $\beta_{x2}^{att}$ between categories 3 and 2, and $\beta_{x3}^{att}$ between categories 4 and 3.

As explained in the previous sections, the effect and dummy coding constraints are not only imposed on the attribute effects, but also on the constants and the predictor effects in the regression model for first choices and rankings, on the constants in the regression model for ratings, and on the intercepts and covariate effects in the regression model for the latent classes.

## 2.9 Known-Class Indicator

Sometimes, one has a priori information – for instance, from an external source – on the class membership of some individuals. For example, in a four-class situation, one may know that case 5 belongs to latent class 2 and case 11 to latent class 3. Similarly, one may have a priori information on which class cases do not belong to. For example, again in a four-class situation, one may know that case 19 does not belong to latent class 2 and that case 41 does not belong to latent classes 3 or 4. In Latent GOLD, there is an option – called "*Known Class*" – for indicating to which latent classes cases do *not* belong to.

Let $\boldsymbol{\tau}_i$ be a vector of 0-1 variables containing the "*Known Class*" information for case $i$, where $\tau_{ix} = 0$ if it is known that case $i$ does not belong to class $x$, and $\tau_{ix} = 1$ otherwise. The vector $\boldsymbol{\tau}_i$ modifies the model with covariates defined in equation (4) as follows:

$$P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\tau}_i) = \sum_{x=1}^{K} \tau_{ix}\, P(x|\mathbf{z}_i^{cov}) \prod_{t=1}^{T_i} P(y_{it}|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred}).$$

As a result of this modification, the posterior probability of belonging to class $x$ will be equal to 0 if $\tau_{ix} = 0$.

The known-class option has three important applications.

1. It can be used to estimate models with training cases; that is, cases for which class membership has been determined using a gold standard method. Depending on how this training information is obtained, the missing data mechanism will be MCAR (Missing Completely At Random, where the known-class group is a random sample from all cases), MAR (Missing At Random, where the known-class group is a random sample given observed responses and covariate values), or NMAR (Not

Missing At Random, where the known-class group is a non-random sample and thus may depend on class membership itself). MAR occurs, for example, in clinical applications in which cases with more than a certain number of symptoms are subjected to further examination to obtain a perfect classification (diagnosis). NMAR may, for example, occur if training cases that do not belong to the original sample under investigation are added to the data file.

Both in the MAR and MCAR situation, parameter estimates will be unbiased. In the NMAR situation, however, unbiased estimation requires that separate class sizes are estimated for training and non-training cases (McLachlan and Peel, 2000). This can easily be accomplished by expanding the model of interest with a dichotomous covariate that takes on the value 0 for training cases and 1 for non-training cases.

2. Another application is specifying models with a partially missing discrete variable that affects one or more response variables. An important example is the *complier average causal effect* (CACE) model proposed by Imbens and Rubin (1997), which can be used to determine the effect of a treatment conditional on compliance with the treatment. Compliance is, however, only observed in the treatment group, and is missing in the control group. This CACE model can be specified as a model in which class membership (compliance) is known for the treatment group, and which a treatment effect is specified only for the compliance class.

3. The known-class indicator can also be used to specify *multiple-group LC models*. Suppose we have a three-class model and two groups, say males and females. A multiple-group LC model is obtained by indicating that there are six latent classes, were males may belong to classes 1–3 and females to classes 4–6. To get the correct output, the grouping variable should not only be used as the known-class indicator, but also as a nominal covariate.

## 2.10 Zero-Inflated Models

When the zero-inflated option is used, the model is expanded with $M$ latent classes that are assumed to respond with probability one to a certain category; that is, $P(y_{it} = m|x, \mathbf{z}_{it}^{pred}) = 1$ for $x = K + m$. Such latent classes are

sometimes referred to as stayer classes (in mover-stayer models) or brand-loyal classes (in brand-loyalty models).

## 2.11 Restrictions on the Regression Coefficients

Various types of restrictions can be imposed on the Class-specific regression coefficients: attribute and predictor effects can be fixed to zero, restricted to be equal across certain or all Classes, and constrained to be ordered. Moreover, the effects of numeric attributes can be fixed to one. These constraints can either be used as a priori restrictions derived from theory or as post hoc restrictions on estimated models.

Certain restrictions apply to parameters within each Class, while others apply across Classes. The within-Class restrictions are:

- No Effect: the specified effect(s) are set to zero;
- Offset: the selected effect(s) are set to one, thus serving as an offset.[2] The offset effect applies to numeric attributes only.

Between-Class restrictions are:

- Merge Effects: the effects of a selected attribute/predictor are equated across 2 or more specified Classes;
- Class Independent: the effects of a selected attribute/predictor are equated across all Classes;
- Order (ascending or descending): in each Class, the effect of a selected numeric attribute/predictor is assumed to have the same sign or the effects corresponding to a selected nominal attribute/predictor are assumed to be ordered (either ascending or descending). That is, for numeric attributes/predictors, the ascending restriction implies that the Class-specific coefficients should be at least zero ($\beta \geq 0$) and the descending restriction that they are at most zero ($\beta \leq 0$). For nominal attributes/predictors, ascending implies that the coefficient of category

[2]The term offset stems from the generalized linear modeling framework. It refers to a regression coefficient that is fixed to 1, or equivalently, to a component that offsets the linear part of the regression model by a fixed amount. An offset provides the same role as a cell weight in log-linear analysis. An offset is in fact, the log of a cell weight.

$p+1$ is larger than or equal to the one of category $p$ ($\beta_p \leq \beta_{p+1}$, for each $p$) and descending that the coefficient of category $p+1$ is smaller than or equal to the one of category $p$ ($\beta_p \geq \beta_{p+1}$, for each $p$).

The "Class Independent" option can be used to specify models in which some attribute and predictor effects differ across Classes while others do not. This can either be on a priori grounds or can be based on the test statistics from previously estimated models. More specifically, if the Wald(=) test is not significant, it makes sense to check whether an effect can be assumed to be Class independent.

There is a special variant of the Class-independent option called "No Simple" that can be used in conjunction with the constants in a rating model. With this option, the constants are modeled as $\beta_{xm}^{con} = \beta_{\cdot m}^{con} + \beta_{x\cdot}^{con} \cdot y_m^*$, where $\beta_{x\cdot}^{con}$ is subjected to an effect or dummy coding constraint. This specification of Class-specific constants is much more parsimonious and is, in fact, equivalent to how $x$-$y$ relationships with ordinal $y$'s are modeled in LC Cluster models. Rather that estimating $K \cdot M$ intercept terms, one now estimates only $M + K - 1$ coefficients; that is, one extra coefficient per extra latent class.

"Order" constraints are important if one has a priori knowledge about the sign of an effect. For example, the effect of price on persons' preferences is usually assumed to be negative – or better, non-positive – for each latent class (segment). If the price effect is specified to be "Descending", the resulting parameter estimate(s) will be constrained to be in agreement with this assumption.

The "No Effect" option makes it possible to specify a different regression equation for each latent class. More specifically, each latent class may have different sets of attributes and predictors affecting the choices. Post hoc constraints can be based on the reported z value for each of the coefficients. An example of an a priori use of this constraint is the inclusion of a random-responder class, a class in which all coefficient are zero. This is specified as follows:

| | Class 1 | Class 2 | Class 3 | Class 4 |
|---|---|---|---|---|
| Constants | – | 2 | 3 | 4 |
| Attribute1 | – | 2 | 3 | 4 |
| Attribute2 | – | 2 | 3 | 4 |

,

where "–" indicates that the effect is equal to 0. In this example, Class 1 is the random-responder class.

"Merge Effects" is a much more flexible variant of "Class Independent". It can be used to equate the parameters for any set of latent classes. Besides post hoc constraints, very sophisticated a priori constraints can be imposed with this option. An important application is the specification of LC DFactor structures in which each latent class corresponds to the categories of two or more latent variables. For example, consider a set of constraints of the form:

| | Class 1 | Class 2 | Class 3 | Class 4 | |
|---|---|---|---|---|---|
| Constants | 1 | 1 | 3 | 3 | |
| Attribute1 | 1 | 2 | 1 | 2 | , |
| Attribute2 | 1 | 2 | 1 | 2 | |

where the same numbers in a given row indicate that the associated class parameters are equal. This restricted 4-Class model is a 2-dimensional DFactor model: the categories of DFactor 1 differ with respect to the constants and the categories of DFactor 2 with respect to the two attribute effects. Specifically, level 1 of DFactor 1 is formed by Classes 1 and 2 and level 2 by Classes 3 and 4; level 1 of DFactor 2 is formed by Classes 1 and 3 and level 2 by Classes 2 and 4.

The option "Offset" can be used to specify any nonzero fixed-value constraint on the Class-specific effect of a numeric attribute. This means that it is possible to refine the definition of any Class (segment) by enhancing or reducing the estimated part-worth utility of any numeric attribute for that Class. Recall that numeric attribute $p$ enters as $\beta_{xp}^{att}\, z_{itmp}^{att}$ in the linear part of the conditional logit model. Suppose, after estimating the model, the estimate for $\beta_{xp}^{att}$ turned out to be 1.5 for Class 1. If $z_{itmp}^{att}$ is specified to be an offset, the importance of this attribute to be reduced (1.5 would be reduced to 1) for this Class. But suppose that you wish to enhance the importance of this attribute for Class 1; say, you wish to restrict $\beta_{xp}^{att}$ to be equal to 2. The trick is to recode the attribute, replacing each code by twice the value. Thus, the recoded attribute is defined as $2 \cdot z_{itmp}^{att}$. If we restrict the effect of this recoded attribute to 1, we obtain $1 \cdot 2 \cdot z_{itmp}^{att}$, which shows that the effect of $z_{itmp}^{att}$ is equated to 2. Such recoding can be done easily within Latent GOLD Choice, using the Replace option.

In addition to post hoc refinements to customize the definition of the resulting latent classes, the offset restriction can also be used to make the Classes conform to various theoretical structures. Probably the most important a priori application of "Offset" is that of defining stayer- or brand-loyal classes. A brand-loyal class selects one of the brands with a probability equal

to 1 and is not affected by the other attributes. An example of a restrictions table corresponding to such a structure is:

| | Class 1 | Class 2 | Class 3 | Class 4 |
|---|---|---|---|---|
| Brand1(100) | $*$ | $-$ | $-$ | $-$ |
| Brand2(100) | $-$ | $*$ | $-$ | $-$ |
| Brand3(100) | $-$ | $-$ | $*$ | $-$ |
| Constants | $-$ | $-$ | $-$ | 4 |
| Attribute1 | $-$ | $-$ | $-$ | 4 |
| Attribute2 | $-$ | $-$ | $-$ | 4 |

.

Here, "$-$" means no effect and "$*$" means offset. As can be seen, Classes 1, 2, and 3 are only affected by an offset, and Class 4 – the brand-switching or mover class – is affected by the constants and the two attributes. The numeric "attributes" Brand1(100), Brand2(100), and Brand3(100) are brand "dummies" that take on the value 100 for the brand concerned and are 0 otherwise.[3] As a result of the fixed effect of 100, the probability of selecting the corresponding brand will be equal to one. To illustrate this, suppose that a choice set consists of three alternatives and that (only) alternative 1 is associated with brand 1. The probability that someone belonging to Class 1 selects alternative 1 equals $\exp(100)/[\exp(100) + \exp(0) + \exp(0)] = 1.0000$.[4] Although this model is similar to a zero-inflated model, the offset-based specification is much more flexible in the sense that the number of brand-loyal classes does not need to coincide with the number of alternatives per set. In the above example, the sets could consist of four instead of three alternatives, say three different brands and a "none" alternative.

Now, we will discusses several more advanced applications of the restriction options. The first is a model for ratings with a parsimonious specification of the Class dependence of the constants. Let "One" be an attribute with the constant value l. The model of interest is obtained with the restrictions

[3]It is not necessary to assume that the 100 appears in all alternatives for the brand concerned. The value could also be 100 if a particular condition is fulfilled – for example, if the price of the evaluated product is larger than a certain amount – and 0 otherwise. This shows that the offset option provides a much more flexible way of specifying classes with zero response probabilities than the zero-inflated option.

[4]While a value of 100 for the offset can be used to fix a probability to 1.0000, a value of -100 can be use to fix a probability to 0.0000. For example, $\exp(-100)/[\exp(-100) + \exp(0) + \exp(0)] = 0.0000$.

table

| | Class 1 | Class 2 | Class 3 | Class 4 |
|---|---|---|---|---|
| Constants | 1 | 1 | 1 | 1 |
| One | – | 2 | 3 | 4 |
| Attribute1 | 1 | 2 | 3 | 4 |
| Attribute2 | 1 | 2 | 3 | 4 |

.

Instead of having a separate set of constants for each latent class, the restricted constant for category $m$ in Class $x$ equals $\beta_{xm}^{con} = \beta_m^{con} + y_m^* \cdot \beta_{x1}^{att}$, where for identification, $\beta_{11}^{att} = 0$. Note that this is equivalent to using the "no-simple" setting for the constants and similar to the treatment of ordinal indicators in the LC Cluster and DFactor Modules of Latent GOLD.

Suppose you assume that the effect of price is negative (descending) for Classes 1-3 and unrestricted for Class 4. This can be accomplished by having two copies of the price variable in the model, say Price1 and Price2. The effect of Price1 is specified as ordered and is fixed to zero in Class 4. The effect of Price2 is fixed to zero in Classes 1-3.

Suppose your assumption is that the effect of a particular attribute is at least 2. This can be accomplished by combining a fixed value constraint with an order constraint. More precisely, an additional attribute defined as $2 \cdot z_{itmp}^{att}$ is specified to be an offset and the effect of the original attribute $z_{itmp}^{att}$ defined to be ascending.

Our final example is an exploratory variant of the DFactor structure described above. Suppose you want a two-DFactor model without assumptions on which discrete factor influences which attribute effects. This can be accomplished having 3 copies of all attributes in the attributes file. With two attributes (brand and price), the restriction table is of the form

| | Class 1 | Class 2 | Class 3 | Class 4 |
|---|---|---|---|---|
| Brand1 | 1 | 1 | 1 | 1 |
| Brand2 | – | – | 3 | 3 |
| Brand3 | – | 2 | – | 2 |
| Price1 | 1 | 1 | 1 | 1 |
| Price2 | – | – | 3 | 3 |
| Price3 | – | 2 | – | 2 |

.

The first copy (Brand1 and Price1) defines a main effect for each attribute. The second copy (Brand2 and Price2) is used to define the first DFactor, a contrast between Classes 3/4 and 1/2. The third copy (Brand3 and Price3) specifies DFactor 2 by means of a contrast between Classes 2/4 and 1/3.

## 2.12 General Latent Class Choice Model

In the previous subsections, we described the various elements of the LC model implemented in Latent GOLD Choice. Now, we will combine all these elements and provide the structure of the general LC choice model. The general probability density function associated with case $i$ is

$$\begin{aligned} P(\mathbf{y}_i|\mathbf{z}_i,\boldsymbol{\tau}_i) &= \sum_{x=1}^{K} \tau_{ix}\, P(x|\mathbf{z}_i^{cov}) P(\mathbf{y}_i|x,\mathbf{z}_i^{att},\mathbf{z}_i^{pred}) \\ &= \sum_{x=1}^{K} \tau_{ix}\, P(x|\mathbf{z}_i^{cov}) \prod_{t=1}^{T_i} \left[P(y_{it}|x,\mathbf{z}_{it}^{att},\mathbf{z}_{it}^{pred})\right]^{v_{it}}, \end{aligned} \tag{6}$$

where $P(x|\mathbf{z}_i^{cov})$ and $P(y_{it}=m|x,\mathbf{z}_{it}^{att},\mathbf{z}_{it}^{pre})$ are parameterized by logit models; that is,

$$\begin{aligned} P(x|\mathbf{z}_i^{cov}) &= \frac{\exp(\eta_{x|\mathbf{z}_i})}{\sum_{x'=1}^{K}\exp(\eta_{x'|\mathbf{z}_i})} \\ P(y_{it}=m|x,\mathbf{z}_{it}^{att},\mathbf{z}_{it}^{pre},s_{it}) &= \frac{\exp(s_{it}\cdot\,\eta_{m|x,\mathbf{z}_{it}})}{\sum_{m'\in A_{it}}\exp(s_{it}\,\cdot\eta_{m'|x,\mathbf{z}_{it}})}. \end{aligned}$$

The linear model for $\eta_{x|\mathbf{z}_i}$ is

$$\eta_{x|\mathbf{z}_i} = \gamma_{0x} + \sum_{r=1}^{R}\gamma_{rx} z_{ir}^{cov}.$$

For first choices and rankings, $\eta_{m|x,\mathbf{z}_{it}}$ equals

$$\eta_{m|x,\mathbf{z}_{it}} = \beta_{xm}^{con} + \sum_{p=1}^{P}\beta_{xp}^{att}\, z_{itmp}^{att} + \sum_{q=1}^{Q}\beta_{xmq}^{pre}\, z_{itq}^{pre},$$

if $m \in A_{it}$ and $-\infty$ otherwise. For ratings,

$$\eta_{m|x,\mathbf{z}_{it}} = \beta_{xm}^{con} + y_m^* \cdot \left(\sum_{p=1}^{P}\beta_{xp}^{att}\, z_{itp}^{att} + \sum_{q=1}^{Q}\beta_{xq}^{pre}\, z_{itq}^{pre}\right).$$

# 3 Estimation and Other Technical Issues

## 3.1 Log-likelihood and Log-posterior Function

The parameters of the LC choice model are estimated by means of Maximum Likelihood (ML) or Posterior Mode (PM) methods. The likelihood function

is derived from the probability density function defined in equation (6). Let $\boldsymbol{\vartheta}$ denote the vector containing the $\gamma$ and $\beta$ parameters. As before, $\mathbf{y}_i$ and $\mathbf{z}_i$ denote the vectors of dependent and explanatory variables for case $i$ , and $I$ denotes the total number of cases.

ML estimation involves finding the estimates for $\boldsymbol{\vartheta}$ that maximize the log-likelihood function

$$\log \mathcal{L} = \sum_{i=1}^{I} w_i \log P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta}).$$

Here, $P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})$ is the probability density associated with case $i$ given parameter values $\boldsymbol{\vartheta}$ and $w_i$ is the *Case Weight* corresponding to case $i$.[5] This case weight $w_i$ can be used to group identical response patterns or to specify (complex survey) sampling weights. In the former case, $w_i$ will serve as a frequency count, and in the latter case, Latent GOLD Choice will provide pseudo ML estimates (Patterson, Dayton, and Graubard, 2002).[6] The other type of weight – *Replication Weight* $v_{it}$ – that was introduced in the previous section modifies the definition of the relevant probability density $P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})$. The exact form of $P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})$ is described in equation (6).

In order to prevent boundary solutions or, equivalently, to circumvent the problem of non-existence of ML estimates, we implemented some ideas from Bayesian statistics in Latent GOLD Choice. The boundary problem that may occur is that the (multinomial) probabilities of the model for the latent classes or the model for the choices, rankings, or ratings may converge to zero. This occurs if a $\beta$ or $\gamma$ parameter becomes very extreme, tends to go to (minus) infinity. The boundary problem is circumvented by using Dirichlet priors for the latent and the response probabilities (Clogg et al., 1991; Galindo-Garre, Vermunt, and Bergsma, 2004; Gelman et. al., 1996; Schafer, 1997). These are so-called conjugate priors since they have same form as the corresponding multinomial probability densities. The implication of using priors is that the estimation method is no longer ML but PM (Posterior Mode).

Denoting the assumed priors for $\boldsymbol{\vartheta}$ by $p(\boldsymbol{\vartheta})$ and the posterior by $\mathcal{P}$, PM estimation involves finding the estimates for $\boldsymbol{\vartheta}$ that maximize the log-

[5] In order to simplify the discussion, in this section we discuss only on the situation without known-class indicators.

[6] In Latent GOLD Choice Advanced, there is a more elegant option for dealing with sampling weights, as well as with other complex survey sampling features.

posterior function

$$\begin{aligned}\log \mathcal{P} &= \log \mathcal{L} + \log p(\boldsymbol{\vartheta}) \\ &= \sum_{i=1}^{I} w_i \log P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta}) + \log p(\boldsymbol{\vartheta}),\end{aligned}$$

or, in other words, finding the point where $\frac{\partial \log \mathcal{P}}{\partial \boldsymbol{\vartheta}} = 0$. Algorithms that are used to solve this problem – EM and Newton-Raphson – are described below.

The user-defined parameters in the priors $p(\boldsymbol{\vartheta})$ can be chosen in such a way that $\log p(\boldsymbol{\vartheta}) = 0$, which makes PM estimation turn into ML estimation. PM estimation can also be seen as a form of penalized ML estimation, in which $p(\boldsymbol{\vartheta})$ serves as a function penalizing solutions that are too near to the boundary of the parameter space and, therefore, smoothing the estimates away from the boundary.

## 3.2 Missing Data

### 3.2.1 Dependent variable

If the value of the dependent variable is missing for one or more of the replications of case $i$, the replications concerned are omitted from the analysis. The remaining replications will, however, be used in the analysis. Thus, instead of using list-wise deletion of cases, Latent GOLD Choice provides ML or PM estimates based on all available information. The assumption that is made is that the missing data are missing at random (MAR) or, equivalently, that the missing data mechanism is ignorable (Little and Rubin, 1987; Schafer, 1997; Skrondal and Rabe-Hesketh, 2004; Vermunt, 1997).

In the case of missing data, it is important to clarify the interpretation of the chi-squared goodness-of-fit statistics. Although parameter estimation with missing data is based on the MAR assumption, the chi-squared statistics not only test whether the model of interest holds, but also the much more restrictive MCAR (missing completely at random) assumption (see Vermunt, 1997). Thus, caution should be used when interpreting the overall goodness-of-fit tests in situations involving missing data.

### 3.2.2 Attributes, predictors, and covariates

Missing values on attributes will never lead to exclusion of cases or replications from the analysis. If the technical option for including missing values

on covariates and predictors is off, cases with missing covariate values and replications with missing predictor values are excluded from the analysis. When this technical option is on, such cases and replications are retained by imputing the missing values using the method described below.

Missing values on numeric predictors and covariates are replaced by the sample mean. This is the mean over all cases without a missing value for covariates and the mean of all replications without a missing value for predictors. Missing values on numeric attributes are not imputed with a mean, but with a 0, which implies that a missing value in the alternatives file is, in fact, equivalent to using a 0.

Missing values on nominal attributes, predictors, and covariates is dealt with via the design matrix. In fact, the effect is equated to zero for the missing value category. Recall the effect and dummy coding schemes illustrated in subsection 2.8 for the case of a nominal attribute with 4 categories. Suppose there is also a missing category. In the case of effects coding, the design matrix that is set up for the 3 non-redundant terms is then

$$\begin{array}{lrrr} \text{category 1} & 1 & 0 & 0 \\ \text{category 2} & 0 & 1 & 0 \\ \text{category 3} & 0 & 0 & 1 \\ \text{category 4} & -1 & -1 & -1 \\ \text{missing} & 0 & 0 & 0 \end{array} .$$

As can be seen, the entries corresponding to the missing category are all equal to 0, which amounts to setting its coefficient equal to zero. Since in effect coding the unweighted mean of the coefficients equals zero, equating the effect of the missing value category to zero implies that it is equated to the unweighted average of the effects of the other four categories. This imputation method for nominal variables is therefore similar to mean imputation with numeric variables.

In the case of dummy coding with the first category as the reference category, the design matrix that is set up for the 3 non-redundant terms is

$$\begin{array}{lrrr} \text{category 1} & 0 & 0 & 0 \\ \text{category 2} & 1 & 0 & 0 \\ \text{category 3} & 0 & 1 & 0 \\ \text{category 4} & 0 & 0 & 1 \\ \text{missing} & 1/4 & 1/4 & 1/4 \end{array} .$$

The number 1/4 (one divided by the number of categories of the nominal attribute concerned) implies that the parameter of the missing value category is equated to the unweighted mean of the parameters of the other four categories. Note that the coefficient for the reference category is fixed to 0. Also with "dummy last", we would get a row with 1/4s for the missing value category.

## 3.3 Prior Distributions

The different types of priors have in common that their user-defined parameters (*Bayes Constants*) denoted by $\alpha$ can be interpreted as adding $\alpha$ observations – for instance, the program default of one – generated from a conservative null model (as is described below) to the data. All priors are defined in such a way that if the corresponding $\alpha$'s are set equal to zero, $\log\, p\,(\boldsymbol{\vartheta}) = 0$, in which case we will obtain ML estimates. We could label such priors as "non-informative". Below we present the $\log\, p\,(\boldsymbol{\vartheta})$ terms for the various types of distributions without their normalizing constants. The symbols $U^{cov}$ and $U^{att,pre}$ are used to denote the number of different covariate and attribute/predictor patterns. A particular pattern is referred to by the index $u$.

The Dirichlet prior for the latent probabilities equals

$$\log p\,[P(x|\mathbf{z}_u^{cov})] = \frac{\alpha_1}{K \cdot U^{cov}} \log P(x|\mathbf{z}_u^{cov}).$$

Here, $K$ denotes the number of latent classes and $\alpha_1$ the *Bayes Constant* to be specified by the user. As can be seen, the influence of the prior is equivalent to adding $\frac{\alpha_1}{K}$ cases to each latent class. These cases are distributed evenly over the various covariate patterns. This prior makes the sizes of the latent classes slightly more equal and the covariate effects somewhat smaller.

For the dependent variable, we use the following Dirichlet prior:

$$\log p\left[P(y = m|x, \mathbf{z}_u^{att}, \mathbf{z}_u^{pred})\right] = \frac{\widehat{\pi}_m\, \alpha_2}{K \cdot U^{att,pred}} \log P(y|x, \mathbf{z}_u^{att}, \mathbf{z}_u^{pred}),$$

where $\widehat{\pi}_m$ is the observed marginal distribution of the dependent variable $y$. This prior can be interpreted as adding $\frac{\alpha_2}{K}$ observations to each latent class with preservation of the observed distribution of $y$, where $\alpha_2$ is a parameter to be specified by the user. The $\frac{\alpha_2}{K}$ observations are distributed evenly over the observed attribute/predictor patterns. This prior makes the class-specific

response probabilities slightly more similar to each other and smooths the $\beta$ parameters somewhat towards zero.

The influence of the priors on the final parameter estimates depends on the values chosen for the $\alpha$'s, as well as on the sample size. The default settings are $\alpha_1 = \alpha_2 = 1.0$. This means that with moderate sample sizes the influence of the priors on the parameter estimates is negligible. Setting $\alpha_1 = \alpha_2 = 0$ yields ML estimates.

## 3.4 Algorithms

To find the ML or PM estimates for the model parameters $\boldsymbol{\vartheta}$, Latent GOLD Choice uses both the EM and the Newton-Raphson algorithm. In practice, the estimation process starts with a number of EM iterations. When close enough to the final solution, the program switches to Newton-Raphson. This is a way to exploit the advantages of both algorithms; that is, the stability of EM when it is far away from the optimum and the speed of Newton-Raphson when it is close to the optimum.

The task to be performed for obtaining PM estimates for $\boldsymbol{\vartheta}$ is finding the parameter values for which

$$\frac{\partial \log \mathcal{P}}{\partial \boldsymbol{\vartheta}} = \frac{\partial \log \mathcal{L}}{\partial \boldsymbol{\vartheta}} + \frac{\partial \log p(\boldsymbol{\vartheta})}{\partial \boldsymbol{\vartheta}} = 0. \tag{7}$$

Here,

$$\begin{aligned}
\frac{\partial \log \mathcal{L}}{\partial \boldsymbol{\vartheta}} &= \sum_{i=1}^{I} w_i \frac{\partial \log P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})}{\partial \boldsymbol{\vartheta}} \\
&= \sum_{i=1}^{I} w_i \frac{\partial \log \sum_{x=1}^{K} P(x|\mathbf{z}_i^{cov}, \boldsymbol{\vartheta}) P(\mathbf{y}_i|x, \mathbf{z}_i^{att}, \mathbf{z}_i^{pred}, \boldsymbol{\vartheta})}{\partial \boldsymbol{\vartheta}} \\
&= \sum_{i=1}^{I} \sum_{x=1}^{K} w_{xi} \frac{\partial \log P(x|\mathbf{z}_i^{cov}, \boldsymbol{\vartheta}) P(\mathbf{y}_i|x, \mathbf{z}_i^{att}, \mathbf{z}_i^{pred}, \boldsymbol{\vartheta})}{\partial \boldsymbol{\vartheta}},
\end{aligned} \tag{8}$$

where

$$w_{xi} = w_i \, P(x|\mathbf{z}_i, \mathbf{y}_i, \boldsymbol{\vartheta}) = w_i \, \frac{P(x|\mathbf{z}_i^{cov}, \boldsymbol{\vartheta}) P(\mathbf{y}_i|x, \mathbf{z}_i^{att}, \mathbf{z}_i^{pred}, \boldsymbol{\vartheta})}{P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})}. \tag{9}$$

The *EM algorithm* is a general method for dealing with ML estimation with missing data (Dempster, Laird, and Rubin, 1977; McLachlan and Krishnan, 1997). This method exploits the fact that the first derivatives of the

incomplete data log-likelihood ($\log \mathcal{L}$) equal the first derivatives of the complete data log-likelihood ($\log \mathcal{L}^c$). The complete data is the log-likelihood that we would have if we knew to which latent class each case belongs:

$$\begin{aligned}
\log \mathcal{L}^c &= \sum_{i=1}^{I}\sum_{x=1}^{K} w_{xi} \log P(x|\mathbf{z}_i^{cov}, \boldsymbol{\vartheta}) P(\mathbf{y}_i|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred}, \boldsymbol{\vartheta}) \\
&= \sum_{i=1}^{I}\sum_{x=1}^{K} w_{xi} \log P(x|\mathbf{z}_i^{cov}, \ \boldsymbol{\vartheta}) \\
&\quad + \sum_{i=1}^{I}\sum_{x=1}^{K} w_{xi} \sum_{t=1}^{T_i} v_{it} \log P(y_{it}|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred}, \boldsymbol{\vartheta}).
\end{aligned} \tag{10}$$

Each $\nu$th cycle of the EM algorithm consist of two steps. In the Expectation (E) step, estimates $\widehat{w}_{xi}^{\nu}$ are obtained for $w_{xi}$ via equation (9) filling in $\widehat{\boldsymbol{\vartheta}}^{\nu-1}$ as parameter values. The Maximization (M) step, involves finding new $\widehat{\boldsymbol{\vartheta}}^{\nu}$ improving $\log \mathcal{L}^c$. Note that, actually, we use PM rather than ML estimation, which means that in the M step we update the parameters in such a way that

$$\log \mathcal{P}^c = \log \mathcal{L}^c + \log p(\boldsymbol{\vartheta}) \tag{11}$$

increases rather than (10). Sometimes closed-form solutions are available in the M step. In other cases, standard iterative methods can be used to improve the complete data log-posterior defined in equation (11). Latent GOLD Choice uses iterative proportional fitting (IPF) and unidimensional Newton in the M step (see Vermunt 1997, Appendices).

Besides the EM algorithm, we also use a *Newton-Raphson* (NR) method. [7] In this general optimization algorithm, the parameters are updated as follows:

$$\widehat{\boldsymbol{\vartheta}}^{\nu} = \widehat{\boldsymbol{\vartheta}}^{\nu-1} - \varepsilon \ \boldsymbol{H}^{-1}\mathbf{g}.$$

The gradient vector $\mathbf{g}$ contains the first-order derivatives of the log-posterior to all parameters evaluated at $\widehat{\boldsymbol{\vartheta}}^{\nu-1}$, $\mathbf{H}$ is the Hessian matrix containing the second-order derivatives to all parameters, and $\varepsilon$ is a scalar denoting the step size. Element $g_k$ of $\mathbf{g}$ equals

$$g_k = \sum_{i=1}^{I} w_i \frac{\partial \log P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})}{\partial \vartheta_k} + \frac{\partial \log p(\boldsymbol{\vartheta})}{\partial \vartheta_k},$$

[7] Haberman (1988) proposed estimating standard LC models by Newton Raphson.

and element $H_{kk'}$ of $\mathbf{H}$ equals

$$H_{kk'} = \sum_{i=1}^{I} w_i \frac{\partial^2 \log P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})}{\partial \vartheta_k \partial \vartheta_{k'}} + \frac{\partial^2 \log p(\boldsymbol{\vartheta})}{\partial \vartheta_k \partial \vartheta_{k'}}.$$

Latent GOLD Choice computes these derivatives analytically. The step size $\varepsilon$ ($0 < \varepsilon \leq 1$) is needed to prevent decreases of the log-posterior to occur. More precisely, when a standard NR update $-\mathbf{H}^{-1}\mathbf{g}$ yields a decrease of the log-likelihood, the step size is reduced till this no longer occurs.

The matrix $-\mathbf{H}^{-1}$ evaluated at the final $\widehat{\boldsymbol{\vartheta}}$ yields the standard estimate for the asymptotic variance-covariance matrix of the model parameters: $\widehat{\Sigma}_{standard}(\boldsymbol{\vartheta}) = -\widehat{\mathbf{H}}^{-1}$.[8] Latent GOLD Choice also implements two alternative estimates for $\Sigma(\boldsymbol{\vartheta})$. The first alternative is based on the outer-product of the cases' contributions to the gradient vectors; that is, $\widehat{\Sigma}_{outer}(\boldsymbol{\vartheta}) = \widehat{\mathbf{B}}^{-1}$, where element $B_{kk'}$ of $\mathbf{B}$ is defined as

$$B_{kk'} = \frac{N}{N-1} \sum_{i=1}^{I} w_i \frac{\partial \log P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})}{\partial \vartheta_k} \frac{\partial \log P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta})}{\partial \vartheta_{k'}}.$$

Note that $\mathbf{B}$ is the sample covariance matrix of the case-specific contributions to the elements of the gradient vector.

The third estimator for $\Sigma(\boldsymbol{\vartheta})$ is the so-called robust, sandwich, or Huber-White estimator, which is defined as

$$\widehat{\Sigma}_{robust}(\boldsymbol{\vartheta}) = \widehat{\mathbf{H}}^{-1}\,\widehat{\mathbf{B}}\;\widehat{\mathbf{H}}^{-1}.$$

The advantage of $\widehat{\Sigma}_{outer}(\boldsymbol{\vartheta})$ compared to the other two is that is much faster to compute because it uses only first derivatives. It may thus be an alternative for $\widehat{\Sigma}_{standard}(\boldsymbol{\vartheta})$ in large models. The advantage of the robust method is that contrary to the other two methods, it does not rely on the assumption that the model is correct.

Note that $\widehat{\Sigma}(\boldsymbol{\vartheta})$ can be used to obtain the standard error for any function $h(\widehat{\boldsymbol{\vartheta}})$ of $\widehat{\boldsymbol{\vartheta}}$ by the delta method:

$$\widehat{se}\left(h(\widehat{\boldsymbol{\vartheta}})\right) = \sqrt{\left(\frac{h(\widehat{\boldsymbol{\vartheta}})}{\partial \widehat{\boldsymbol{\vartheta}}}\right)' \widehat{\Sigma}(\boldsymbol{\vartheta}) \left(\frac{h(\widehat{\boldsymbol{\vartheta}})}{\partial \widehat{\boldsymbol{\vartheta}}}\right)}. \tag{12}$$

[8] The matrix $-\mathbf{H}$ is usually referred to as the observed information matrix, which serves as an approximation of the expected information matrix.

Latent GOLD Choice uses the delta method, for example, to obtain standard errors of probabilities and redundant parameters.

Inequality restrictions – needed for ordered clusters, order-restricted predictor effects, and positive variances – are dealt with using an active-set variant of the Newton-Raphson method described above (Galindo-Garre, Vermunt, Croon, 2001; Gill, Murray, and Wright, 1981). For that purpose, the effects involved in the order constraints are reparameterized so that they can be imposed using simple nonnegativity constraints of the form $\vartheta \geq 0$. In an active-set method, the equality constraint associated with an inequality constraint becomes activate if it is violated (here, parameter is equated to 0 if it would otherwise become negative), but remains inactive if its update yields an admissible value (here, a positive update).

## 3.5 Convergence

The exact algorithm implemented in Latent GOLD Choice works as follows. The program starts with EM until either the maximum number of EM iterations ( *Iteration Limits EM*) or the EM convergence criterion (*EM Tolerance*) is reached. Then, the program switches to NR iterations which stop when the maximum number of NR iterations (*Iteration Limits Newton-Raphson*) or the overall converge criterion (*Tolerance*) is reached. The convergence criterion that is used is

$$\sum_{u=1}^{npar} \left| \frac{\widehat{\vartheta}_u^{\nu} - \widehat{\vartheta}_u^{\nu-1}}{\widehat{\vartheta}_u^{\nu-1}} \right|,$$

which is the sum of the absolute relative changes in the parameters. The program also stops its iterations when the change in the log-posterior is negligible, i.e., smaller than $10^{-12}$.

The program reports the iteration process in Iteration Detail. Thus, it can easily be checked whether the maximum number of iterations is reached without convergence. In addition, a warning is given if one of the elements of the gradient is larger than $10^{-3}$.

It should be noted that sometimes it is more efficient to use only the EM algorithm, which is accomplished by setting *Iteration Limits Newton-Raphson* = 0 in the Technical Tab. This is, for instance, the case in models with many parameters. With very large models, one may also consider suppressing the computation of standard errors and Wald statistics, or to Pause the model estimation to examine preliminary output.

## 3.6 Start Values

Latent GOLD Choice generates random start values. So long as the technical option *Seed* equals 0 (the default option), these differ every time that a model is estimated because the seed of the random number generator is then obtained from the system time. The seed used by the program is reported in the output. A run can be replicated by specifying the reported best start seed as *Seed* in the Technical Tab and setting the number of *Random Sets* to zero.

Since the EM algorithm is extremely stable, the use of random starting values is generally good enough to obtain a converged solution. However, there is no guarantee that such a solution is also the global PM or ML solution. A well-known problem in LC analysis is the occurrence of local maxima which also satisfy the likelihood equations given in (7).

The best way to prevent ending up with a local solution is to use multiple sets of starting values which may yield solutions with different log-posterior values. In Latent GOLD Choice, the use of such multiple sets of random starting values is automated. The user can specify how many sets of starting values the program should use by changing the *Random Sets* option in the Technical Tab. Another relevant parameter is *Iterations* specifying the number of iterations to be performed per start set. More precisely, within each of the random sets, Latent GOLD Choice performs the specified number of EM iterations. Subsequently, within the best 10 percent in terms of log-posterior, the program performs an extra 2 times *Iterations* EM iterations. Finally, it continues with the best solution until convergence. It should be noted that while such a procedure increases considerably the probability of finding the global PM or ML solution, especially if both parameters are set large enough, there is no guarantee that it will be found in a single run.

When a model contains two or more latent classes or one or more DFactors, the starting values procedure will generate the specified number of starting sets and perform the specified number of iterations per set. In one-class models in which local maxima may occur – for example, in models with continuous factors (see Advanced option) – both the specified number of starting sets and iterations per set are reduced by a factor of three. In one-class models in which local maxima cannot occur, the number of starting sets is automatically equated to 1.

With the option *Tolerance*, one can specify the EM convergence criterion to be used within the random start values procedure. Thus, start values

iterations stop if either this tolerance or the maximum number of iterations is reached.

## 3.7 Bootstrapping the P Value of $L^2$ or -2$LL$ Difference

Rather than relying on the asymptotic p value, it also possible to estimate the p value associated with the $L^2$ statistic by means of a parametric bootstrap. This option is especially useful with sparse tables (Langeheine, Pannekoek, and Van de Pol, 1996) and with models containing order restrictions (Galindo and Vermunt, 2005; Vermunt, 1999, 2001). The model of interest is then not only estimated for the sample under investigation, but also for $B$ replication samples. These are generated from the probability distribution defined by the ML estimates. The estimated bootstrap p value, $\widehat{p}_{boot}$, is defined as the proportion of bootstrap samples with a larger $L^2$ than the original sample. The standard error of $\widehat{p}_{boot}$ equals $\sqrt{\frac{\widehat{p}_{boot}(1-\widehat{p}_{boot})}{B}}$. The precision of $\widehat{p}_{boot}$ can be increased by increasing the number of replications $B$. The number of replications is specified by the parameter *Replications*.

A similar procedure is used to obtain a bootstrap estimate of the p value corresponding to the difference in log-likelihood value between two nested models, such as two models with different numbers of latent classes. The -2$LL$-difference statistic is defined as $-2 \cdot (LL_{H_0} - LL_{H_1})$, where $H_0$ refers to the more restricted hypothesized model (say a $K$–class model) and $H_1$ to the more general model (say a model with $K + 1$ classes). Replication samples are generated from the probability distribution defined by the ML estimates under $H_0$. The estimated bootstrap p value, $\widehat{p}_{boot}$, is defined as the proportion of bootstrap samples with a larger -2$LL$-difference value than the original sample.

The bootstrap of the -2$LL$-difference statistic comparing models with different numbers of latent classes was used by McLachlan and Peel (2000) in the context of mixture of normals. Vermunt (2001) used bootstrap p values for both the $L^2$ and the -2$LL$-difference statistic in the context of order-restricted latent class models, where the $L^2$ measured the goodness-of-fit for an ordinal latent class model and the -2$LL$ difference concerned the difference between an order-restricted and an unrestricted latent class model.

The other parameter is *Seed*, which can be used to replicate a bootstrap. The seed used by the bootstrap to generate the data sets is reported in the output.

Two technical details about the implementation of the bootstrap should be mentioned. For each bootstrap replication, the maximum likelihood estimates serve as start values. Thus, no random sets are used for the replications. To gain efficiency in term of computation time, the iterations within a bootstrap replication terminate when the replicated $L^2$ is smaller (-2$LL$-diff value is larger) than the original one, even if the convergence criterion or the maximum number of iterations is not reached.

## 3.8 Identification Issues

Sometimes LC models are not identified; that is, it may not be possible to obtain unique estimates for some parameters. Non-identification implies that different parameter estimates yield the same log-posterior or log-likelihood value. When a model is not identified, the observed information matrix, $-\mathbf{H}$, is not full rank, which is reported by the program. Another method to check whether a model is identified is to run the model again with different starting values. Certain model parameters are not identified if two sets of starting values yield the same $\log \mathcal{P}$ or $\log \mathcal{L}$ values with different parameter estimates.

With respect to possible non-identification, it should be noted that the use of priors may make models identified that would otherwise not be identified. In such situations, the prior information is just enough to uniquely determine the parameter values.

A related problem is "weak identification", which means that even though the parameters are uniquely determined, sometimes the data is not informative enough to obtain stable parameter estimates. Weak identification can be detected from the occurrence of large asymptotic standard errors. Local solutions may also result from weak identification.

Other "identification issues" are related to the order of the Classes and the uniqueness of parameters for nominal variables. For unrestricted Choice models, the Classes are reordered according to their sizes: the first Class is always the largest Class. Parameters ($\gamma$'s and $\beta$'s) involving nominal variables are identified by using either effect or dummy coding, which means that parameters sum to zero over the relevant indices or that parameters corresponding to the first or last category are fixed to zero. Note that the Parameters output also contains the redundant $\gamma$ and $\beta$ parameters, and in the case of effect coding also their standard errors.

## 3.9 Selecting and Holding out Choices or Cases

The replication and case weights can be used to omit certain choices or cases (records with a common case ID) from the analysis. With a weight equal to zero, one can remove a choice/case from the analysis, and no output is provided for this choice/case. Alternatively, a very small weight (1.0e-100) can be used to exclude choices/cases for parameter estimation, while retaining the relevant prediction and classification output.

### 3.9.1 Replication and case weights equal to zero

Setting case weights equal to zero will eliminate the corresponding cases from the analysis. This feature can be used to select a subset of cases for the analysis. For example, by specifying a variable with the value 1 for males and 0 for females as a case weight, one will perform an analysis for males only. This "zero case weight" option makes it straightforward to perform separate analyses for different subgroups that are in the same data file. It should be noted that no output is provided for the cases with zero weights.

Similarly, with a replication weight equal to zero, one removes the corresponding replication from the analysis. This option can, therefore, be used to select choices to be used for parameter estimation; for example, one may wish to select the first and last choice from a full ranking for a maximum-difference analysis.

### 3.9.2 Replication weights equal to a very small number

An important feature of the Latent GOLD Choice program is that it allows specifying hold-out choices. These are choices that are not used for parameter estimation, but for which one obtains prediction information. Hold-out choices are defined by means of replications weights equal to a very small number; i.e., 1.0e-100. These replications will be excluded when estimating the specified model. Their predicted values, however, may be written to the output file. This "very small replication weight" option can be used for validation purposes; that is, to determine the prediction performance of the estimated model for hold-out choices.

### 3.9.3 Case weights equal to a very small number

In some situations, one may desire removing certain cases from the analysis, but nevertheless obtaining classification and prediction output for all cases. This can be accomplished by using case weights equal to a very small number – i.e., 1.0e-100 – for the cases that should not be used for parameter estimation. The program treats such a weight as if it were a zero, which means that results are not influenced by the presence of these cases and that computation time is comparable to the analysis of a data set without these cases. An important difference with the "zero case weight" option is that this "very small case weight" option yields classification and prediction information for the cases concerned.

One possible application is the analysis of very large data sets. With this option one can use a subset of cases for parameter estimation, but obtain class membership information for all cases. Another application is predicting class membership for new cases based on parameter values obtained with another sample. By appending the new cases to the original data file and giving them a weight equal to 1.0e-100, one obtains the relevant output for these cases after restoring and re-estimating the original model.

# 4 The Latent Gold Choice Output

Below, we provide technical details on the quantities presented in the various Latent GOLD Choice output sections (Model Summary, Parameters, Importance, Profile, ProbMeans, Set Profile, Set Probmeans, Iteration Detail, Frequencies, Standard Classification, and Covariate Classification), as well as on the output that can be written to files (Standard Classification, Covariate Classification, Predicted values, Individual Coefficients, Cook's D, and Variance-Covariance Matrix).

## 4.1 Model Summary

This first part of the output section reports the number of cases ($N = \sum_{i=1}^{I} w_i$), the total number of replications ($N_{rep} = \sum_{i=1}^{I} w_i \sum_{t=1}^{T_i} v_{it}$), the number of estimated parameters ($npar$), the number of activated constraints (in models with order restrictions), the seed used by the pseudo random number generator, the seed of the best start set, and the seed used by the bootstrap procedure.

The last part (Variable Detail) contains information on the variables that are used in the analysis. The other four parts - Chi-squared Statistics, Log-likelihood Statistics, Classification Statistics, Covariate Classification Statistics, and Prediction Statistics - are described in more detail below.

### 4.1.1 Chi-squared statistics

The program reports chi-squared and related statistics, except when the data file contains replication weights other than 0 or 1. The three reported chi-squared measures are the likelihood-ratio chi-squared statistic $L^2$, the Pearson chi-squared statistic $X^2$, and the Cressie-Read chi-squared statistic $CR^2$. Before giving the definitions of the chi-squared statistics, we need to explain two types of groupings that have to be performed with the original cases.

The first is the grouping of identical cases; that is, cases that have the same covariate, known-class, predictor, and attribute values, and give the same responses. This yields $I^*$ unique data patterns with observed frequency counts denoted by $n_{i^*}$, where $i^*$ denotes a particular data pattern. These frequency counts are obtained by summing the case weights $w_i$ of the cases with data pattern $i^*$; that is, $n_{i^*} = \sum_{i \in i^*} w_i$.[9] In order to obtain the chi-squared statistics, we also need to group cases with identical covariate, known-class, predictor, and attribute values, which amounts to grouping cases without taking into account their responses.[10] This yields the sample sizes $N_u$ for the $U$ relevant multinomials, where $u$ denotes a particular multinomial or "covariate" pattern. These sample sizes are obtained by $N_u = \sum_{i \in u} w_i$ or $N_u = \sum_{i^* \in u} n_{i^*}$.[11] Note that $N = \sum_{u=1}^{U} N_u$.

Let $\widehat{m}_{i^*}$ denote the estimated cell count for data pattern $i^*$, which is obtained by:

$$\widehat{m}_{i^*} = N_{u_{i^*}} \widehat{P}(\mathbf{y}_{i^*} | \mathbf{z}_{i^*}), \tag{13}$$

i.e., by the product of the total number of cases with the same "covariate" pattern as data pattern $i^*$ ($N_{u_{i^*}}$) and the estimated multinomial probability

[9] With the somewhat loose but rather simple notation $i \in i^*$ we mean "all the cases with data pattern $i^*$".

[10] With missing values on some replications, also the missing data pattern is used as a grouping criterion. That is, cases belonging to the same "covariate" pattern should also have observed values on the same set of replications or.

[11] With $i \in u$ we mean "all the cases with covariate pattern $u$", and with $i^* \in u$ "all the data patterns with covariate pattern $u$".

corresponding to data pattern $i^*$.[12]

Using these definitions of $\widehat{m}_{i^*}$, $n_{i^*}$, and $N$, the chi-squared statistics are calculated as follows:[13]

$$\begin{aligned} L^2 &= 2\sum_{i^*=1}^{I^*} n_{i^*} \log \frac{n_{i^*}}{\widehat{m}_{i^*}}, \\ X^2 &= \sum_{i^*=1}^{I^*} \frac{(n_{i^*})^2}{\widehat{m}_{i^*}} - N, \\ CR^2 &= 1.8\sum_{i^*=1}^{I^*} n_{i^*} \left[\left(\frac{n_{i^*}}{\widehat{m}_{i^*}}\right)^{2/3} - 1\right]. \end{aligned}$$

The number of degrees of freedom is defined by

$$df = \min\left\{\sum_{u=1}^{U}\left(\prod_{t=1}^{T_u^*} M_{ut}^* - 1\right), N\right\} - npar.$$

Here, $T_u^*$ is the total number of replications in "covariate" pattern $u$, and $M_{ut}^*$ denotes the number of alternatives of the $t$th observed replication corresponding to "covariate" pattern $u$. The term $\min\{\cdot\}$ indicates that $df$ is based on the sample size $N$ when the number of independent cells in the hypothetical frequency table is larger than the sample size. The chi-squared values with the corresponding $df$ yield the asymptotic $p$ -values, which can be used to determine whether the specified model fits the data.

If the *Bootstrap* $L^2$ option is used, the program also provides the estimated bootstrap $p$-value corresponding to the $L^2$ statistic, as well as its standard error. This option is especially useful with sparse tables, in which case the asymptotic $p$-values cannot be trusted. Note that sparseness almost always is a problem in LC choice models. The best indication of sparseness is when $df$ is (much) larger than the total sample size $N$.

The program reports the Bayesian Information Criterion ($BIC$), the Akaike Information Criterion ($AIC$), Akaike Information Criterion 3 ($AIC3$), and the Consistent Akaike Information Criterion ($CAIC$) based on the $L^2$ and $df$, which is the more common formulation in the analysis of frequency

[12] In order to get meaningful chi-squared statistics, in models with a known-class indicator we, in addition, divide by $\sum_{x=1}^{K} \tau_{i^*x} P(x|\mathbf{z}_{i^*})$.

[13] Note that we are using a somewhat unconventional formula for $X^2$. The reason for this is that the sum $\sum_{i^*=1}^{I^*}$ is over the nonzero observed cells only.

tables. They are defined as

$$\begin{aligned} BIC_{L^2} &= L^2 - \log(N)\, df, \\ AIC_{L^2} &= L^2 - 2\ df, \\ AIC3_{L^2} &= L^2 - 3\ df, \\ CAIC_{L^2} &= L^2 - [\log(N) + 1]\ \ df. \end{aligned}$$

These information criteria weight the fit and the parsimony of a model: the lower $BIC$, $AIC$, $AIC3$, or $CAIC$, the better the model.

Use of information criteria based on $L^2$ or $\log \mathcal{L}$ (see below) should yield the same result. The differences between $BIC$, $AIC$, $AIC3$, and $CAIC$ values across models are the same with both methods. However, with extremely large $df$, the $L^2$ based information measures may become more highly negative than the maximum precision can indicate, which makes their rounded values meaningless. In such cases, one has to use the (equivalent) $\log \mathcal{L}$ based measures.

The last statistic that is provided in the chi-squared statistics section is the Dissimilarity Index ($DI$), which is a descriptive measure that is defined as follows:

$$DI = \frac{\left\{ \left( \sum_{i^*=1}^{I^*} |n_{i^*} - \widehat{m}_{i^*}| \right) + \left( N - \sum_{i^*=1}^{I^*} \widehat{m}_{i^*} \right) \right\}}{2\, N}.$$

It should be noted that the term $(N - \sum_{i^*=1}^{I^*} \widehat{m}_{i^*})$ captures the contribution of the zero observed cells to $DI$. This term is added to the formula because $\sum_{i^*=1}^{I^*} |n_{i^*} - \widehat{m}_{i^*}|$ is a sum over the non-zero observed cell counts only. $DI$ is a descriptive measure indicating how much observed and estimated cell frequencies differ from one another. It indicates which proportion of the sample should be moved to another cell to get a perfect fit.

### 4.1.2 Log-likelihood statistics

The program also reports the values of the log-likelihood ($\log \mathcal{L}$ ), the log-prior ($\log p(\boldsymbol{\vartheta})$), and log-posterior ($\log \mathcal{P}$) . Recall that

$$\begin{aligned} \log \mathcal{L} &= \sum_{i=1}^{I} w_i \log \widehat{P}(\mathbf{y}_i | \mathbf{z}_i), \\ \log \mathcal{P} &= \log \mathcal{L} + \log p(\widehat{\boldsymbol{\vartheta}}). \end{aligned}$$

In addition, the Bayesian Information Criterion ($BIC$), the Akaike Information Criterion ($AIC$), the Akaike Information Criterion 3 ($AIC3$),[14] and the Consistent Akaike Information Criterion ($CAIC$) based on the log-likelihood are reported. These are defined as

$$\begin{aligned} BIC_{\log \mathcal{L}} &= -2 \log \mathcal{L} + (\log N)\ npar, \\ AIC_{\log \mathcal{L}} &= -2 \log \mathcal{L} + 2\ npar, \\ AIC3_{\log \mathcal{L}} &= -2 \log \mathcal{L} + 3\ npar, \\ CAIC_{\log \mathcal{L}} &= -2 \log \mathcal{L} + [(\log N) + 1]\ npar. \end{aligned}$$

If the *Bootstrap -2LL diff* option is used, the program also provides the estimated bootstrap $p$-value (and the standard error) for the -2$LL$ difference test between a restricted and an unrestricted model.

### 4.1.3 Classification statistics

This set of statistics contains information on how well the observed $y$ and $z$ values predict the latent class, or, in other words, how well the latent classes are separated. Classification is based on the latent classification or posterior class membership probabilities. For response pattern $i$, these are calculated as follows:

$$\widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i) = \frac{\widehat{P}(x|\mathbf{z}_i^{cov})\widehat{P}(\mathbf{y}_i|x, \mathbf{z}_i^{att}, \mathbf{z}_i^{pred})}{\widehat{P}(\mathbf{y}_i|\mathbf{z}_i)}. \tag{14}$$

These quantities are used to compute the estimated proportion of classifications errors ($E$), as well as three $R^2$-type measures for nominal variables: the proportional reduction of classification errors $R^2_{x,errors}$, a measure based on entropy labelled $R^2_{x,entropy}$, and a measure based on qualitative variance labelled $R^2_{x,variance}$. The latter is similar to the Goodman and Kruskal tau-b association coefficient for nominal dependent variables (Magidson, 1981).

The proportion of classification errors is defined as:

$$E = \frac{\sum_{i=1}^{I} w_i \left[1 - \max \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)\right]}{N}.$$

[14] New results by Andrews and Currim (2003) and Dias (2004) suggest that AIC3 is a better criterion than BIC and AIC in determining the number of latent classes in choice models.

Each of the three $R^2$-type measures is based on the same type of reduction of error structure; namely,

$$R_x^2 = \frac{\text{Error}(x) - \text{Error}(x|\mathbf{z}, \mathbf{y})}{\text{Error}(x)}, \tag{15}$$

where $\text{Error}(x)$ is the total error when predicting $x$ without using information on $\mathbf{z}$ and $\mathbf{y}$, and $\text{Error}(x|\mathbf{z}, \mathbf{y})$ is the prediction error if we use all observed information from the cases. $\text{Error}(x|\mathbf{z}, \mathbf{y})$ is defined as the (weighted) average of the case-specific errors $\text{Error}(x|\mathbf{z}_i, \mathbf{y}_i)$,

$$\text{Error}(x|\mathbf{z}, \mathbf{y}) = \frac{\sum_{i=1}^{I} w_i \text{Error}(x|\mathbf{z}_i, \mathbf{y}_i)}{N}.$$

The three $R^2$ measures differ in the definition of $\text{Error}(x|\mathbf{z}_i, \mathbf{y}_i)$. In $R^2_{x,errors}$, it equals $1 - \max \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)$, in $R^2_{x,entropy}$, $\sum_{x=1}^{K} - \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i) \log \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)$, and in $R^2_{x,variance}$, $1 - \sum_{x=1}^{K} [\widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)]^2$. In the computation of the total error $\text{Error}(x)$, the $\widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)$ are replaced by the estimated marginal latent probabilities $\widehat{P}(x)$, which are defined as

$$\widehat{P}(x) = \frac{\sum_{i=1}^{I} w_i \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)}{N} = \frac{\sum_{i=1}^{I} \widehat{w}_{xi}}{N}. \tag{16}$$

The Average Weight of Evidence ($AWE$) criterion adds a third dimension to the information criteria described above. It weights fit, parsimony, and the performance of the classification (Banfield and Raftery, 1993). This measure uses the so-called classification log-likelihood, which is equivalent to the complete data log-likelihood $\log \mathcal{L}^c$, i.e.,

$$\log \mathcal{L}^c = \sum_{i=1}^{I} \sum_{x=1}^{K} \widehat{w}_{xi} \log \widehat{P}(x|\mathbf{z}_i^{cov}) \widehat{P}(\mathbf{y}_i|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred}).$$

$AWE$ can now be defined as

$$AWE = -2 \log \mathcal{L}^c + 2 \left( \frac{3}{2} + \log N \right) npar.$$

The lower $AWE$, the better a model.

The Classification Table cross-tabulates modal and probabilistic class assignments. More precisely, the entry $(x, x')$ contains the sum of the class $x$

posterior membership probabilities for the cases allocated to modal class $x'$. Hence, the diagonal elements ($x = x'$) are the numbers of correct classifications per class and the off-diagonal elements ($x \neq x'$) the corresponding numbers of misclassifications. From the classification table, one can not only see how many cases are misclassified (as indicated by the proportion of classification errors $E$), but also detect which are the most common types of misclassifications. If a particular entry $(x, x')$ with $x \neq x'$ is large, this means that classes $x$ and $x'$ are not well separated.

The marginals of the Classification Table provides the distribution of cases across classes under modal (column totals) and probabilistic (row totals) classification. Except for very rare situations, these marginal distributions will *not* be equal to one another. This illustrates the phenomenon that modal class assignments do not reproduce the estimated latent class distribution. Whereas the row totals are in agreement with the estimated classes sizes,[15] the column totals provide the latent class distribution that is obtained when writing the class assignments to a file using the Latent GOLD Choice output-to-file option.

### 4.1.4 Covariate classification statistics

These statistics indicate how well one can predict class membership from an individual's covariate values, and are therefore only of interest if the estimated model contains active covariates. The measures are similar to the ones that are reported in the section "Classification Statistics"; that is, the estimated proportion of classification errors, the proportional reduction of classification errors, an entropy-based $R^2$ measure, and a qualitative variance-based $R^2$ measure. The difference is that now the predictions (and computations) are based on the model probabilities $\widehat{P}(x|\mathbf{z}_i)$ instead of the posterior probabilities $\widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)$. Whereas the total error can still be denoted as Error($x$), the model prediction error in equation (15) should now be denoted as Error($x|\mathbf{z}$) instead of Error($x|\mathbf{z}, \mathbf{y}$).

### 4.1.5 Prediction statistics

Prediction statistics indicate how well the observed choices, rankings, or ratings are predicted by the specified model. For rankings, the prediction

[15] There may be a very small difference, which is caused by the Bayes constant for the latent classes.

statistics are based on first choices only. For choice and rating variables, all replications are used for obtaining the prediction measures.

The predicted values used in the computation of the prediction statistics are based on the estimated individual-specific response probabilities, which are denoted by $\widehat{P}_{m|it}$. For ratings, we also make use of the estimated expected values $\widehat{y}_{it} = \sum_{m=1}^{M} y_m^* \widehat{P}_{m|it}$, where $y_m^*$ is the score for response category $m$. As is shown in detail below, $\widehat{P}_{m|it}$ is computed by weighting Class-specific estimates by the posterior membership probabilities $\widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)$. This means that our procedure can be called *posterior-mean/mode or expected/modal a posteriori prediction* .

The individual-specific response probabilities $\widehat{P}_{m|it}$ can be obtained as follows:

$$\widehat{P}_{m|it} = \sum_{x=1}^{K} \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)\widehat{P}(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred}). \tag{17}$$

As can be seen, these are weighted averages of the Class-specific response probabilities, where the posterior class-membership probabilities serve as weights.

There are two other prediction methods – HB-like and marginal mean prediction. In the first, one obtains $\widehat{P}_{m|it}$ with the individual-specific utilities $\widehat{\eta}_{m|it}$,

$$\widehat{P}_{m|it} = \frac{\exp(\widehat{\eta}_{m|it})}{\sum_{m'=1}^{M} \exp(\widehat{\eta}_{m'|it})} \tag{18}$$

The $\widehat{V}_{itm}$ are weighted averages of the Class-specific utilities defined in equation (2), where the posterior class-membership probabilities serve as weights; that is,

$$\widehat{\eta}_{m|it} = \sum_{x=1}^{K} \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)\, \widehat{\eta}_{m|x,\mathbf{z}_{it}}.$$

Because of the similarity with prediction in Hierarchical Bayes (HB) procedures, we call this alternative method *HB-like prediction.* Note that the way we compute $\widehat{\eta}_{m|it}$ is equivalent to computing $\widehat{\eta}_{m|it}$ with the individual-specific $\widehat{\beta}_{ip}$ parameters defined in equation (20).

*Marginal mean (mode) prediction* differs from posterior mean prediction in that the prior class membership probabilities $\widehat{P}(x|\mathbf{z}_i)$ are used in the formula for $\widehat{P}_{m|it}$ given in equation (17) instead of the posterior membership probabilities $\widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)$. Whereas posterior mean and HB-like prediction provide a good indication of the within-sample prediction performance,

marginal mean prediction gives a good indication of the out–of-sample prediction performance.

The most natural predicted value for a categorical dependent variable is the mode; that is, the $m$ with the largest $\widehat{P}_{m|it}$. The *Prediction Table* cross-classifies observed and predicted values based on such a modal assignment. For ratings, which are ordinal dependent variables, we make use of the mean ($\widehat{y}_{it}$) in some of the error measures. Error measures may also be based on the estimated probabilities instead of a single predicted value.

The error measures reported in prediction statistics are obtained as follows:

$$\text{Error} = \frac{\sum_{i=1}^{I} w_i \sum_{t=1}^{T_i} v_{it}\, \text{Error}_{it}}{\sum_{i=1}^{I} w_i \sum_{t=1}^{T_i} v_{it}} \tag{19}$$

As can be seen, "Error" is a weighted average of the replication-specific errors "Error$_{it}$". Latent GOLD Choice uses four types of error measures ( *Squared Error, Absolute Error, Minus Log-likelihood, and Prediction Error)*, which differ in the definition of Error$_{it}$. For ratings, the Error$_{it}$ for *Squared Error* and *Absolute Error* equal $(y_{it} - \widehat{y}_{it})^2$ and $|y_{it} - \widehat{y}_{it}|$, respectively. For choices and ranking, these errors equal $\sum_{m=1}^{M}[I_m(y_{it}) - \widehat{P}_{m|it}]^2$ and $\sum_{m=1}^{M}|I_m(y_{it}) - \widehat{P}_{m|it}|$, where indicator variable $I_m(y_{it})$ equals 1 if $y_{it} = m$ and 0 otherwise. The Error$_{it}$ for *Minus Log-likelihood* equals $\sum_{m=1}^{M} - I_m(y_{it}) \ln \widehat{P}_{m|it}$. In the computation of *Prediction Error,* Error$_{it}$ equals 0 if the modal prediction is correct and 1 otherwise.

The general definition of the (pseudo) $R^2$ of an estimated model is the reduction of errors compared to the errors of a baseline model. More precisely,

$$R_y^2 = \frac{\text{Error(baseline)} - \text{Error(model)}}{\text{Error(baseline)}}.$$

Latent GOLD Choice uses two different baseline models, called *Baseline* and *Baseline(0)*, yielding two $R^2$ measures, called $R_y^2$ and $R_y^2(0)$. In *Baseline,* the Error is computed with response probabilities equal to the average $\widehat{P}_{m|it}$,

$$\widehat{P}_m = \frac{\sum_{i=1}^{I} w_i \sum_{t=1}^{T_i} v_{it}\, \widehat{P}_{m|it}}{\sum_{i=1}^{I} w_i \sum_{t=1}^{T_i} v_{it}},$$

In models with an unrestricted set of constants, $\widehat{P}_m$ equals the observed distribution of $y$. In that case, *Baseline* can be interpreted as the constants only model. The response probabilities under *Baseline(0)* are $\widehat{P}_m(0) = 1/M$, which means that *Baseline(0)* is the equal-probability model.

## 4.2 Parameters

The first part of the Parameters output contains *Class-specific* and *overall* $R_y^2$ and $R_y^2(0)$ values based on *Squared Error*. The overall measures are the same as the ones appearing in *Prediction Statistics*. The logic behind the computation of the Class-specific $R_{y|x}^2$ measures is the same as for the overall measures (see description of *Prediction Statistics*). The Class-specific errors are obtained by

$$\text{Error}_{y|x} = \frac{\sum_{i=1}^{I} w_i \sum_{t=1}^{T_i} v_{it}\, \text{Error}_{xit}}{\sum_{i=1}^{I} w_i \sum_{t=1}^{T_i} v_{it}},$$

with $\widehat{w}_{xi} = w_i\ \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)$, as in equation (9). The definition of $\text{Error}_{xit}$ is based on the Class-specific response probabilities $\widehat{P}(y_{it} = m|x, \mathbf{z}_{it}^{att}, \mathbf{z}_{it}^{pred})$ or, shortly, $\widehat{P}_{m|xit}$. For ratings, the predicted value equals $\widehat{y}_{xit} = \sum_{m=1}^{M} y_m^* P_{m|xit}$ and the corresponding error is $\text{Error}_{xit} = (y_{it} - \widehat{y}_{xit})^2$. For choice and ranking variables, $\text{Error}_{xit}$ equals $\sum_{m=1}^{M}[I_m(y_{it}) - \widehat{P}_{m|xit}]^2$. Similar to the overall $R_y^2$ measures, the *Baseline* error is based on the average $\widehat{P}_{m|xit}$ and *Baseline(0)* on $1/M$.

In the second part of the Parameters output, the program reports the estimates obtained for the $\beta$ and $\gamma$ parameters appearing in the linear predictors $\eta$, the estimates for error variances and covariances $\sigma$, as well as the corresponding estimated asymptotic standard errors, $\widehat{se}(\beta)$, $\widehat{se}(\gamma)$, and $\widehat{se}(\sigma)$. These standard errors are the squared roots of the diagonal elements of the estimated variance-covariance matrix $\widehat{\Sigma}(\boldsymbol{\vartheta})$. As described earlier, one of three methods can be used to obtain $\widehat{\Sigma}(\boldsymbol{\vartheta})$, yielding either a standard, outer-product based, or robust standard errors and Wald statistics.

The significance of sets of parameters can be tested by means of the reported Wald statistic labeled *Wald*. We also report a Wald statistic labeled *Wald(=)*, which tests whether regression coefficients are equal between Classes (Class Independent). The general formula for a Wald statistic ($W^2$) is

$$W^2 = \left(\mathbf{C}'\boldsymbol{\vartheta}\right)' \left(\mathbf{C}'\,\widehat{\Sigma}(\boldsymbol{\vartheta})\mathbf{C}\right)^{-1} \left(\mathbf{C}'\boldsymbol{\vartheta}\right),$$

where the tested set of linear constraints is: $\mathbf{C}'\boldsymbol{\vartheta} = \mathbf{0}$. The Wald test is a chi-squared test. Its number of degrees of freedom equals the number of constraints. Computation of standard errors and Wald statistics can be suppressed, which may be useful in models with many parameters.

The Parameters output also contains the means and standard deviations of the conditional logit coefficients (last two columns in model for

choices/rankings/ratings). These are the typical fixed and random effects in multilevel, mixed, or random-coefficient logit models. Let $\widehat{\beta}_{xp}$ denote the estimated value of one of the conditional logit parameters, which can be a constant, an attribute effect, or a predictor effect. Using basic statistics calculus, the *Mean* of $\widehat{\beta}_{xp}$ can be defined as $\sum_{x=1}^{K} \widehat{P}(x)\,\widehat{\beta}_{xp}$ and the *Std.Dev.* of $\widehat{\beta}_{xp}$ as $\sqrt{\sum_{x=1}^{K} \widehat{P}(x)(\widehat{\beta}_{xp})^2 - \left[\sum_{x=1}^{K} \widehat{P}(x)\,\widehat{\beta}_{xp}\right]^2}$.

## 4.3 Importance

The Importance output reports the maximum effect for each of the attributes, including the constants, as well as re-scaled maximum effects that add up to one within latent classes.

Let $a$ denote a level of attribute $p$, $A_p$ its total number of levels, and $\widehat{\eta}_{a|xp}$ the utility associated with level $a$ for latent class $x$. For numeric attributes, $\widehat{\eta}_{a|xp}$ equals the attribute effect times the numeric score of category $a$ ($z_{ap}\beta^{att}_{xp}$); for nominal attributes, it is simply the effect for category $a$ ($\beta^{att}_{xap}$). The maximum effect of attribute $p$ for latent class $x$ is defined as

$$\text{maxeff}_{xp} = \max(\widehat{\eta}_{a|xp}) - \min(\widehat{\eta}_{a|xp}).$$

These maximum effects can be compared both across attributes and across latent classes.

Often it is relevant to compare the relative importances of the attributes across latent classes. These relative importances or relative maximum effects are obtained as follows:

$$\text{releff}_{xp} = \frac{\text{maxeff}_{xp}}{\sum_p \text{maxeff}_{xp}}.$$

As can be seen, $\text{releff}_{xp}$ is a maximum effect that is re-scaled to sum to 1 across attributes within a latent class. The relative importances are depicted in a plot. Attributes can be deleted from the Importance output using the plot control. The relative effects are then rescaled to sum to one for the remaining attributes. This feature can, for example, be useful if one is interested in relative effects without considering the constants or the effect corresponding to a none option.

## 4.4 Profile and ProbMeans

The content of the Profile and ProbMeans output will be explained together because these two output sections are strongly related. Both sections contain 1) marginal latent probabilities, 2) transformed Class-specific attribute effects, and 3) information on the relation between (active and inactive) covariates and class membership.

The first row of each output section contains the estimated marginal latent class probabilities $\widehat{P}(x)$ (see equation 16). In Profile these are called *Class Size* and in ProbMeans *Overall Probability*.

The Profile output contains transformed attributes effects ($\beta$ parameters), including the constants. As above, let $a$ denote a level of the attribute $p$, $A_p$ its total number of levels, and $\widehat{\eta}_{a|xp}$ the utility associated with level $a$ for latent class $x$. The reported "choice probabilities" for attribute $p$ are obtained as follows:

$$\widehat{P}_p(a|x) = \frac{\exp(\widehat{\eta}_{a|xp})}{\sum_{a'=1}^{A} \exp(\widehat{\eta}_{a'|xp})}.$$

The $\widehat{P}_p(a|x)$ can be interpreted as the estimated choice probabilities in a set of $A_p$ alternatives that differ only with respect to the attribute concerned. For numeric attributes, we also report the means $\sum_{a=1}^{A_p} z_{ap}\widehat{P}_p(a|x)$.

In ProbMeans, the choice probabilities $\widehat{P}_p(a|x)$ are re-scaled to sum to one over latent classes. That is,

$$\widehat{P}_p(x|a) = \frac{\widehat{P}(x)\widehat{P}_p(a|x)}{\sum_{x'=1}^{K} \widehat{P}(x')\widehat{P}_p(a|x')}.$$

This number can be interpreted as the probability of being in latent class $x$ given choice $a$ on "set" $p$.

The third part of the Profile and ProbMeans output sections provides information for covariates. This is information obtained by aggregating and re-scaling posterior membership probabilities (Magidson and Vermunt, 2001). Let $b$ denote a particular level of covariate $r$,and $B_r$ the number of categories of the covariate concerned, and let the frequency count $\widehat{n}_r(x, b)$ be defined as follows:

$$\widehat{n}_r(x, b) = \sum_{i:z_{ir}=b} w_i \, \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i),$$

where $i : z_{ir} = b$ denotes that the sum is over the cases with value $b$ on the covariate concerned. In Profile, we report the probability of being in

covariate level $b$ given that one belongs to latent class $x$,

$$\widehat{P}_r(b|x) = \frac{\widehat{n}_r(x,b)}{\sum_{b'=1}^{B_r} \widehat{n}_r(x,b')},$$

and for numeric covariates also the means $\sum_{b=1}^{B} z_{br}\widehat{P}_r(b|x)$, where $z_{br}$ is the score of covariate category $b$. ProbMeans contains the probability of being in latent class $x$ given covariate level $b$:

$$\widehat{P}_r(x|b) = \frac{\widehat{n}_r(x,b)}{\sum_{x'=1}^{K} \widehat{n}_r(x',b)}$$

For nominal attributes/covariates, the Profile plot depicts the choice probabilities $\widehat{P}_p(a|x)$ and covariate probabilities $\widehat{P}_r(b|x)$. For numeric attributes and covariates, the Profile plot contains 0-1 means, which are means that are re-scaled to be in the 0-1 interval. In ProbMeans, the quantities $\widehat{P}_p(x|a)$ and $\widehat{P}_r(b|x)$ are plotted in Uni- and Tri-plots (Magidson and Vermunt, 2001; Vermunt and Magidson 2000). Similar plots have been proposed by Van der Ark and Van der Heijden (1998) and Van der Heijden, Gilula, and Van der Ark (1999) for standard LC and latent budget models.

A nice feature of the Profile and ProbMeans output is that it describes the relationships between the latent variable and all variables selected as attributes or covariates. This means that even if a certain covariate effect is fixed to zero, one still obtains its ProbMeans information. This feature is exploited in the "inactive covariates method". Advantages of working with inactive instead of active covariates are that the estimation time is not increased and that the obtained solution is the same as without covariates.

## 4.5 Set Profile and Set ProbMeans

The Set Profile and Set ProbMeans output sections contain information on the estimated choice probabilities per choice set. For rankings, these are based on the first choice replications only. For choices and ratings, all replications are used.

Let $\ell$ denote a particular choice set number as indicated by the *Set ID* variable. The Class-specific and the overall choice probabilities for *Set* $\ell$ are obtain as follows:

$$\widehat{P}_\ell(m|x) \;=\; \frac{\sum_{i=1}^{I} \widehat{w}_{xi} \sum_{t\in\ell} v_{it} \widehat{P}(y_{it}=m|x,\mathbf{z}_{it}^{att},\mathbf{z}_{it}^{pred})}{\sum_{i=1}^{I} \widehat{w}_{xi} \sum_{t\in\ell} v_{it}},$$

$$\widehat{P}_\ell(m) \;=\; \frac{\sum_{i=1}^{I} w_i \sum_{t\in\ell} v_{it}\widehat{P}_{m|it}}{\sum_{i=1}^{I} w_i \sum_{t\in\ell} v_{it}}.$$

Here, $\widehat{w}_{xi}$ is the case weight times the posterior membership probability (see equation 9), and $\widehat{P}_{m|it}$ is the individual-specific choice probability which, depending on the type of prediction, is defined by equation (17) or (18). The computation of the *Set Average* is the same, except that the summations are over all $t$ instead of $t \in \ell$.

Set Profile also contains information on the observed choice probabilities $p_\ell(m)$, as well as residuals per alternative and per set that compare observed with overall estimated choice probabilities. The standardized residual (*StdResid*) for alternative $m$ of set $\ell$ is obtained as follows:

$$\frac{p_\ell(m) - \widehat{P}_\ell(m)}{\sqrt{\widehat{P}_\ell(m)}}\sqrt{N_\ell},$$

where $N_\ell = \sum_i w_i \sum_{t\in\ell} v_{it}$. The univariate residual (*UniResid*) for set $\ell$ is defined as:

$$\frac{\sum_{m=1}^{M_\ell} \frac{\left[p_\ell(m)-\widehat{P}_\ell(m)\right]^2}{\widehat{P}_\ell(m)} N_\ell}{M_\ell - 1.}.$$

Note that this is just a Pearson chi-squared divided by the number of "degrees of freedom", or the number of possible alternatives in set $\ell$ minus 1.

The Set Probmeans is obtained by re-scaling the $\widehat{P}_\ell(m|x)$; that is,

$$\widehat{P}_\ell(x|m) = \frac{\widehat{P}(x)\widehat{P}_\ell(m|x)}{\sum_{m=1}^{M_\ell} \widehat{P}(x)\widehat{P}_\ell(m|x)}.$$

These quantities, which can be plotted with the Probmeans in the Uni-plot and Tri-plot, indicate the probability of being in latent class $x$ given that alternative $m$ was selected in set $\ell$.

The file in which the choice sets are defined may contain choice sets that are not presented to respondents. For such "simulation sets", the Set Profile output reports $\widehat{P}(y = m|x, \mathbf{z}_\ell^{att}, \overline{\mathbf{z}}^{pred})$, the estimated Class-specific choice probabilities given their attribute values and the mean of the predictor values.[16] The overall choice probabilities for simulation sets are weighted averages of the Class-specific choice probabilities, where $P(x)$ serves as weight.

[16] Note that the predictor values are missing for simulation sets.

## 4.6 Frequencies / Residuals

Latent GOLD Choice reports estimated and observed cell frequencies ($\widehat{m}_{i^*}$ and $n_{i^*}$), as well as standardized residuals ($\widehat{r}_{i^*}$). The computation of estimated cell entries was described in equation (13). The standardized residuals are defined as

$$\widehat{r}_{i^*} = \frac{\widehat{m}_{i^*} - n_{i^*}}{\sqrt{\widehat{m}_{i^*}}}.$$

Note that $(\widehat{r}_{i^*})^2$ is cell $i^*$'s contribution to the $X^2$ statistic.

This output section also contains a column *Cook's D (Cook's Distance)*. This measure can be used to detect influential cases or, more precisely, cases having a larger influence on the parameter estimates than others. The exact formula that is used in Latent GOLD Choice 4.0 is given in equation (21). A typical cut-point for Cook's D is four times the number of parameters divided by the number of cases (Skrondal and Rabe-Hesketh, 2004). Note that the reported value in a particular row corresponds to the Cook's D for each of the cases with that data pattern.

## 4.7 Classification Information

The Classification output section contains the classification information for each data pattern $i^*$. We report the posterior class membership probabilities $\widehat{P}(x|\mathbf{z}_{i^*}, \mathbf{y}_{i^*})$, as well as the modal Class (the latent class with largest probability). This method of class assignment is sometimes referred to as posterior mode, empirical Bayes modal (EBM), or modal a posteriori (MAP) estimation (Skrondal and Rabe-Hesketh, 2004).

Classification can also be based on covariates only. This involves using the model probabilities $\widehat{P}(x|\mathbf{z}_u)$ – sometimes referred to as prior probabilities – as classification probabilities for each covariate pattern $u$. The same modal classification rule can be applied as with the posterior class membership probabilities.

## 4.8 Output-to-file Options

Five types of items can be written to output files: classification, classification based on covariates, predicted values, individual-specific coefficients, and the estimated variance-covariance matrix of the model parameters.

With *Standard Classification* and *Covariate Classification*, the output file will contain the posterior class-membership probabilities $\widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)$ and the model probabilities $\widehat{P}(x|\mathbf{z}_i)$, respectively, as well as the modal Class assignment based on these probabilities. With the option *Predicted Values* to a file, one obtains the estimated individual-specific choice probabilities $\widehat{P}_{m|it}$ which, depending on the type of prediction, are defined by equation (17) or (18), as well as the predicted value, which is a mode with choices and rankings and a mean with ratings. In addition, a CHAID (.chd) input file can be created for further profiling of the latent classes. (see Section 4.9).

With *Individual Coefficients*, one obtains the estimated individual-specific regression coefficients. Let $\widehat{\beta}_{xp}$ denote the estimated value of one of the conditional logit parameters, which can be a constant, an attribute effect, or a predictor effect. The posterior-mean or expected a posteriori estimate of a particular regression coefficient for case $i$ is defined as follows:

$$\widehat{\beta}_{ip} = \sum_{x=1}^{K} \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)\, \widehat{\beta}_{xp} \tag{20}$$

that is, as a weighted average of the Class-specific coefficients. These estimates are similar to the individual coefficients obtained in multilevel, mixed, random-effects, or hierarchical Bayes (HB) models. The person-specific coefficients can be used to predict person $i$'s new choices.The person-specific coefficients can be used to predict person $i$'s new responses. The posterior standard deviations are defined as

$$\widehat{\sigma}_{\widehat{\beta}_{iq}} = \sqrt{\sum_{x=1}^{K} \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)\, \left(\widehat{\beta}_{xq} - \widehat{\beta}_{iq}\right)^2}$$

Another output-to-file item is *Cook's D (Cook's Distance)*. It can be used to detect influential cases or, more precisely, cases with a large influence on the parameter estimates. The formula that is used the following:

$$C_i = -2\, \mathbf{g}_i'\, \mathbf{H}^{-1}\, \mathbf{g}_i, \tag{21}$$

where $\mathbf{H}$ is the Hessian matrix and $\mathbf{g}_i$ the vector with the gradient contributions of case $i$. A typical cut-point for Cook's D is four times the number of parameters divided by the number of cases (Skrondal and Rabe-Hesketh, 2004).

The last output-to-file item is the *Variance-Covariance Matrix* of the model parameters. Dependent of the type of variance estimator that is requested this will be $\widehat{\boldsymbol{\Sigma}}_{standard}(\boldsymbol{\vartheta})$ , $\widehat{\boldsymbol{\Sigma}}_{outer}(\boldsymbol{\vartheta})$, or $\widehat{\boldsymbol{\Sigma}}_{robust}(\boldsymbol{\vartheta})$. Note that also the variances and covariances involving the omitted categories of the effect coded nominal variables are reported.

## 4.9 The CHAID Output Option

(This option requires the SI-CHAID 4.0 program)

The CHAID (CHi-squared Automatic Interaction Detector) analysis option can be used to assess the statistical significance of each Covariate in its relationship to the latent classes, as well as to develop detailed profiles of these classes, based on the relationships in 3- and higher-way tables. For example, in tutorial #6A, a CHAID analysis is used to explore the relationship between an individual's banking usage during some period (number of checks written, ATM usage, average balance) and the latent classes obtained in tutorial #6. If this option is selected, at the conclusion of the Latent GOLD Choice run, a CHAID (.chd) file is created which can be used as input to the SI-CHAID 4.0 program.

# Part II: Advanced Model Options, Technical Settings, and Output Sections

## 5 Introduction to Part II (Advanced Models)

This part of the manual describes the three Advanced options of Latent GOLD Choice 4.0. These are:

1. An option for specifying models with continuous latent variables, which are referred to as continuous factors (CFactors).

2. A multilevel extension of the LC Choice model, which is a model containing group-level continuous latent variables (GCFactors) and/or a group-level nominal latent variable (GClasses).

3. An option to deal with the sampling design, which yields correct statistical tests for complex survey sampling designs that deviate from simple random sampling.

The *Continuous Factors* (CFactors) option makes it possible to specify random-coefficients conditional logit models. One may, however, also combine CFactors and latent classes in a single model, yielding LC Choice models in which the alternative-specific constants, predictor effects, and/or attribute effects may vary within latent classes.

The *Multilevel Model* option can be used to define LC Choice models for nested data, such as employees nested within firms, pupils nested within schools, clients nested within stores, patients nested within hospitals, citizens nested within regions, and repeated measurements nested within individuals. Note that a LC Choice model is itself a model for two-level data; that is, a model for multiple responses per case. The multilevel LC Choice model is thus, in fact, a model for three-level data; that is, for multiple responses nested within cases and cases nested within groups. As in any multilevel analysis, the basic idea of a multilevel LC Choice analysis is that one or more parameters of the model of interest is allowed to vary across groups using a random-effects modeling approach. In Latent GOLD Choice, the group-level

random effects can either be specified to be continuous (group-level continuous factors: GCFactors) or discrete (group-level latent classes: GClasses), yielding either a parametric or a nonparametric approach, respectively.

One variant of the multilevel LC model involves including group-level random effects in the model for the latent classes, which is a way to take into account that groups differ with respect to the distribution of their members across latent classes (Vermunt, 2003, 2005; Vermunt and Magidson, 2005). Not only the intercept, but also the covariate effects may have a random part. Another variant involves including GCFactors and GClasses in the model for the choices. By combining group-level with case-level latent classes, one obtains a three-level conditional logit model with nonparametric random effects, and by combining group-level continuous factors with case-level continuous factors one obtains a standard three-level random-coefficients conditional logit model. The latter is a special case of the three-level generalized linear model (Vermunt, 2002c, 2004).

The *Survey* option makes it possible to get correct statistical tests with stratified and clustered samples, as well as with sampling weights and samples from finite populations. The design-corrected variance-covariance matrix of the model parameters is obtained by the well-known linearization estimator. Sampling weights can also be dealt with using a two-step procedure that involves estimating the model without sampling weights, and subsequently correcting the latent class distribution and covariate effects using the sampling weights.

The next three sections describe the three Advanced options in more detail. Attention is paid to model components, estimation issues, and application types. The last section discusses the output obtained with the Latent GOLD Choice Advanced options.

# 6 Continuous Factors

## 6.1 Model Components and Estimation Issues

Let $F_{di}$ denote the score of case $i$ on continuous latent variable, factor, or random effect number $d$. The total number of CFactors is denoted by $D$ – thus, $1 \leq d \leq D$ – and the full vector of CFactor scores by $\mathbf{F}_i$ . The maximum number of CFactors that can be included in a Latent GOLD Choice model is three, thus $0 \leq D \leq 3$.

Recall that without CFactors the most general Latent GOLD Choice structure for $P(\mathbf{y}_i|\mathbf{z}_i)$ equals

$$P(\mathbf{y}_i|\mathbf{z}_i) = \sum_{x=1}^{K} P(x|\mathbf{z}_i)P(\mathbf{y}_i|x,\mathbf{z}_i)$$

where

$$P(\mathbf{y}_i|x,\mathbf{z}_i) = \prod_{t=1}^{T_i} P(y_{it}|x,\mathbf{z}_{it}^{att},\mathbf{z}_{it}^{pred}).$$

If we include CFactors in a model, the assumed structure for $P(\mathbf{y}_i|\mathbf{z}_i)$ becomes

$$P(\mathbf{y}_i|\mathbf{z}_i) = \sum_{x=1}^{K} \int_{\mathbf{F}_i} f(\mathbf{F}_i)\, P(x|\mathbf{z}_i)P(\mathbf{y}_i|x,\mathbf{z}_i,\mathbf{F}_i)\, \mathrm{d}\mathbf{F}_i, \tag{22}$$

where

$$P(\mathbf{y}_i|x,\mathbf{z}_i,\mathbf{F}_i) = \prod_{t=1}^{T_i} P(y_{it}|x,\mathbf{z}_{it}^{att},\mathbf{z}_{it}^{pred},\mathbf{F}_i)$$

The $F_{di}$ are assumed to be standard normally distributed and mutually independent. In other words, $f(\mathbf{F}_i) = N(\mathbf{0},\mathbf{I})$, where $\mathbf{I}$ is the identity matrix. As will be shown below, this specification is much less restrictive than one may initially think.

It is also possible to define models – standard random-effects conditional logit models – containing CFactors but no latent classes $x$. That simplifies the structure for $P(\mathbf{y}_i|\mathbf{z}_i)$ to

$$P(\mathbf{y}_i|\mathbf{z}_i) = \int_{\mathbf{F}_i} f(\mathbf{F}_i)\, P(\mathbf{y}_i|\mathbf{z}_i,\mathbf{F}_i)\, \mathrm{d}\mathbf{F}_i$$

with

$$P(\mathbf{y}_i|\mathbf{z}_i,\mathbf{F}_i) = \prod_{t=1}^{T_i} P(y_{it}|\mathbf{z}_{it}^{att},\mathbf{z}_{it}^{pred},\mathbf{F}_i)$$

Equation (22) shows that the $F_{di}$ may appear in the model for the choices, but not in the model for the latent classes.[17] Compared to models without CFactors, the linear predictor in the model for the choices is the expanded with the following additional term:

$$\sum_{d=1}^{D} \lambda_{xmd}^{con} \cdot F_{di} + \sum_{d=1}^{D}\sum_{p=1}^{P} \lambda_{xpd}^{att} \cdot F_{di} \cdot z_{mitp}^{att} + \sum_{d=1}^{D}\sum_{q=1}^{Q} \lambda_{xmqd}^{pred} \cdot F_{di} \cdot z_{itq}^{pred}$$

[17] There is a trick for including CFactor effects in the model for the latent classes using the multilevel option.

In the first term, the $F_{di}$ define random effects for the alternative-specific constants, and the $F_{di} \cdot z^{att}_{mitp}$ and $F_{di} \cdot z^{pred}_{itq}$ product terms define random coefficients for the attributes and predictors. An important difference with the more standard specification of random-effects models is that here each $F_{di}$ can serve as a random effect for each of the model effects, which, as will be shown below, yields parsimonious random-effects covariance structures. Another important difference is that the size of the parameters associated with the random effects may differ across latent classes.

**Model restrictions** One can use the parameter constraints "Class Independent", "No Effect", and "Merge Effects", which imply equal $\lambda$'s among all Classes, zero $\lambda$'s in selected Classes, and equal $\lambda$'s in selected Classes, respectively.

**ML (PM) estimation and technical settings** The main complication in the ML (PM) estimation of models with CFactors is that we have to deal with the multidimensional integral appearing in the definition of the marginal density $P(\mathbf{y}_i|\mathbf{z}_i)$ (see equation 22). Because a closed form expression for this integral is not available, it must be solved using approximation methods. Latent GOLD Choice approximates the conditional density $P(\mathbf{y}_i|\mathbf{z}_i)$ by means of Gauss-Hermite numerical integration, implying that the multidimensional integral is replaced by multiple sums (Bock and Aitkin, 1981). With three CFactors and $B$ quadrature nodes per dimension, the approximate density equals

$$P(\mathbf{y}_i|\mathbf{z}_i) \approx \sum_{x=1}^{K} \sum_{b_1=1}^{B} \sum_{b_2=1}^{B} \sum_{b_3=1}^{B} P(x|\mathbf{z}_i)\, P(\mathbf{y}_i|x, \mathbf{z}_i, F_{b_1}, F_{b_2}, F_{b_3})\, P_{b_1}\, P_{b_2}\, P_{b_3}.$$

Here, $F_{b_d}$ is the location and $P_{b_d}$ the weight corresponding to quadrature node $b_d$ for CFactor $d$. These nodes and weights are obtained from published quadrature tables (Stroud and Secrest, 1966). As can be seen, because of the multiple sums, this approximate density is very similar to the density of a LC model with multiple latent variables. The above approximation also shows that – given the fact that one will usually use at least 10 quadrature points per dimension (Lessafre and Spiessens, 2001) – because of computation burden, it does not make sense to have models with more than three CFactors.

Similar to what Latent GOLD Choice does for standard LC Choice models, the ML (PM) estimation problem for models with CFactors is solved

using a combination of EM and Newton-Raphson with analytic first- and second-order derivatives.

The only new technical setting in models with CFactors is the parameter specifying the number of quadrature nodes to be used in the numerical integration. The default value is 10, the minimum 2, and the maximum 50.

## 6.2 Application Types

### 6.2.1 Random-effects conditional logit models

An important application of the CFactor option involves random-effects discrete-choice modeling (McFadden and Train, 2000; Skrondal and Rabe-Hesketh, 2004).[18] Let us first look at the *random intercept* case in a model for first choices containing $M-1$ alternative-specific constants and $P$ attributes.[19] Such a model has the following form:

$$\eta_{m|\mathbf{z}_{it},F_{1i}} = \beta_m^{con} + \sum_{p=1}^{P} \beta_p^{att}\, z_{itmp}^{att} + \lambda_{m1}^{con} \cdot F_{1i} \tag{23}$$

Note that a single CFactor is used to capture the variation in each of the $M-1$ constants, a specification that is also used in the random-effects multinomial logistic regression model proposed by Hedeker (2003). The random part of the alternative-specific constant corresponding to category $m$ is denoted as $\Psi_m^{con}$. Its variance equals $\sigma^2_{\Psi_m^{con}} = \left(\lambda_{m1}^{con}\right)^2$ and the covariance between $\Psi_m^{con}$ and $\Psi_{m'}^{con}$ equals $\sigma_{\Psi_m^{con},\Psi_{m'}^{con}} = \lambda_{m1}^{con} \cdot \lambda_{m'1}^{con}$.

The model can be expanded to include *random slopes* or *random coefficients* in addition to random intercept terms. However, a slight complication is that one has to decide whether the various random effects should be uncorrelated or not. For uncorrelated random effects, expanding the model of equation (23) with a random slope for the first attribute yields

$$\eta_{m|\mathbf{z}_{it},\mathbf{F}_i} = \beta_m^{con} + \sum_{p=1}^{P} \beta_p^{att}\, z_{itmp}^{att} + \lambda_{m1}^{con} \cdot F_{1i} + \lambda_{12}^{att} \cdot F_{2i} \cdot z_{mit1}^{att}.$$

The variances of the random intercept terms and for the random slope of $z_{mit1}^{att}$ equal $\sigma^2_{\Psi_m^{con}} = \left(\lambda_{xm1}^{con}\right)^2$ and $\sigma^2_{\Psi_1^{att}} = \left(\lambda_{12}^{att}\right)^2$, respectively.

[18] Random-effects models are also referred to as multilevel, hierarchical, mixed-effects, mixed, and random-coefficients models.

[19] Note that in Choice the intercept terms are referred to as constants.

The same model, but now with correlated random effects can be defined as follows:

$$\eta_{m|\mathbf{z}_{it},\mathbf{F}_i} = \beta_m^{con} + \sum_{p=1}^{P} \beta_p^{att}\, z_{itmp}^{att} + \lambda_{m1}^{con} \cdot F_{1i} + \lambda_{11}^{att} \cdot F_{1i} \cdot z_{mit1}^{att} + \lambda_{12}^{att} \cdot F_{2i} \cdot z_{mit1}^{att},$$

As can be seen, here $F_{1i}$ does not only affect the constants but also the effect of $z_{mit1}^{att}$. The variance-covariance matrix of the random effects ($\mathbf{\Sigma_\Psi}$), can be obtained by $\mathbf{\Sigma_\Psi} = \mathbf{\Lambda}\,\mathbf{\Lambda}'$, where $\mathbf{\Lambda}$ is a matrix collecting the $\lambda$ parameters. More specifically, in our example, $\sigma^2_{\Psi_m^{con}} = (\lambda_{m1}^{con})^2$, $\sigma^2_{\Psi_1^{att}} = (\lambda_{11}^{att})^2 + (\lambda_{12}^{att})^2$, $\sigma_{\Psi_m^{con},\Psi_{m'}^{con}} = \lambda_{m1}^{con} \cdot \lambda_{m'1}^{con}$, and $\sigma_{\Psi_m^{con},\Psi_1^{att}} = \lambda_{m1}^{con} \cdot \lambda_{11}^{att}$.

Whereas the random-effects models presented thus far contained as many CFactors as random terms, this is not necessary in general. In fact, with three CFactors – the Latent GOLD Choice maximum – one can define models with any number of random effects. This is accomplished with the following "factor-analytic" specification:

$$\eta_{m|\mathbf{z}_{it},\mathbf{F}_i} = \beta_m^{con} + \sum_{p=1}^{P} \beta_p^{att}\, z_{itmp}^{att} + \sum_{d=1}^{D} \lambda_{xmd}^{con} \cdot F_{di} + \sum_{d=1}^{D}\sum_{p=1}^{P} \lambda_{xpd}^{att} \cdot F_{di} \cdot z_{mitp}^{att}. \quad (24)$$

where again $\mathbf{\Sigma_\Psi} = \mathbf{\Lambda}\,\mathbf{\Lambda}'$. This "factor-analytic" specification in which each CFactor may be associated with multiple random effects is equivalent to the generalized random coefficient (GRC) formulation proposed by Skrondal and Rabe-Hesketh (2004, p. 101). In fact, it is assumed that the unobserved heterogeneity in the regression coefficients can be summarized by at most three underlying CFactors.

### 6.2.2 LC (FM) regression models with random effects

A unique feature of Latent GOLD Choice is that it allows you to combine random effects with latent classes. More specifically, it is possible to specify LC Choice models in which the intercept and/or some of the regression coefficients vary within latent classes. Lenk and DeSarbo (2000) proposed using random effects in FMs of generalized linear models and Böckenholt (2001) proposed using random effects in LC models for ranking data.

It has been observed that the solution of a LC Choice analysis may be strongly affected by heterogeneity in the constants. In *choice-based conjoint studies*, for example, it is almost always the case that respondents differ with respect to their brand preferences, irrespective of the attributes of the

offered products. A LC Choice model captures this brand heterogeneity phenomenon via Classes with different constants. However, the analyst often likes to find relatively small number of latent classes that differ in more meaningful ways with respect to attribute effects on the choices. By including random alternative-specific constants (intercepts) in the LC Choice model, for example,

$$\eta_{m|x,\mathbf{z}_{it},F_{1i}} = \beta^{con}_{xm} + \sum_{p=1}^{P} \beta^{att}_{xp}\, z^{att}_{mitp} + \lambda^{con}_{xm} \cdot F_{1i},$$

it is much more likely that one will succeed in finding such meaningful Classes (segments). The random intercept term, which may have a different effect in each latent class, will filter out (most of) the "artificial" variation in the constants.

# 7 Multilevel LC Choice Model

## 7.1 Model Components and Estimation Issues

To explain the multilevel LC model implemented in Latent GOLD Choice, we need to introduce and some new terminology. Higher-level observations will be referred to as groups and lower-level observations as cases. The records of cases belonging to the same group are connected by the *Group ID* variable. It should be noted that higher-level observations can also be individuals, for example, in longitudinal applications. "Cases" would then be the multiple time points within individuals and replications the multiple choices of an individual at the various time points.

The index $j$ is used to refer to a particular group and $I_j$ to denote the number of cases in group $j$. With $y_{jit}$ we denote the response at replication $t$ of case $i$ belonging to group $j$, with $\mathbf{y}_{ji}$ the full vector of responses of case $i$ in group $j$, and with $\mathbf{y}_j$ the responses of all cases in group $j$. Rather than expanding the notation with new symbols, group-level quantities will be referred to using a superscript $g$: Group-level classes (GClasses), group-level continuous factors (GCFactors), and group-level covariates (GCovariates) are denoted by $x^g$, $\mathbf{F}^g_j$, and $\mathbf{z}^g_j$, and group-level parameters by $\gamma^g$, $\beta^g$, and $\lambda^g$.

The most general probability structure for a multilevel LC Choice model

is

$$P(\mathbf{y}_j|\mathbf{z}_j,\mathbf{z}_j^g) = \sum_{x^g=1}^{K^g} \int_{\mathbf{F}_j^g} f(\mathbf{F}_j^g)\, P(x^g|\mathbf{z}_j^g)\, P(\mathbf{y}_j|\mathbf{z}_j,x^g,\mathbf{F}_j^g)\, \mathrm{d}\mathbf{F}_j^g, \tag{25}$$

where

$$P(\mathbf{y}_j|\mathbf{z}_j,x^g,\mathbf{F}_j^g) = \prod_{i=1}^{I_j} P(\mathbf{y}_{ji}|\mathbf{z}_{ji},x^g,\mathbf{F}_j^g).$$

Assuming that the model of interest may also contain CFactors, for each case $i$, $P(\mathbf{y}_{ji}|\mathbf{z}_{ji},x^g,\mathbf{F}_j^g)$ has a structure similar to the one described in equation (22); that is,

$$P(\mathbf{y}_{ji}|\mathbf{z}_{ji},x^g,\mathbf{F}_j^g) = \sum_{x=1}^{K} \int_{\mathbf{F}_{ji}} f(\mathbf{F}_{ji})\, P(x|\mathbf{z}_{ji},x^g,\mathbf{F}_j^g)\, P(\mathbf{y}_{ji}|x,\mathbf{z}_{ji},\mathbf{F}_{ji},x^g,\mathbf{F}_j^g)\, \mathrm{d}\mathbf{F}_{ji}.$$

where

$$P(\mathbf{y}_{ji}|x,\mathbf{z}_{ji},\mathbf{F}_{ji},x^g,\mathbf{F}_j^g) = \prod_{t=1}^{T_i} P(\mathbf{y}_{jih}|x,\mathbf{z}_{jit}^{att},\mathbf{z}_{jit}^{pred},\mathbf{F}_{ji},x^g,\mathbf{F}_j^g)$$

These four equations show that a multilevel LC Choice model is a model

- for $P(\mathbf{y}_j|\mathbf{z}_j,\mathbf{z}_j^g)$, which is the marginal density of all responses in group $j$ given all exogenous variable information in group $j$,
- containing GClasses ($x^g$) and/or (at most three mutually independent) GCFactors ($\mathbf{F}_j^g$),
- containing GCovariates $\mathbf{z}_j^g$ affecting the group classes $x^g$,
- assuming that the $I_j$ observations for the cases belonging to group $j$ are independent of one another given the GClasses and GCFactors,
- allowing the GClasses and GCFactors to affect the case-level latent classes $x$ and/or the responses $\mathbf{y}_{ji}$.

GCFactors enter in exactly the same manner in the linear term of the conditional logit model as case-level CFactors. We refer to their coefficients as $\lambda_{mxd}^{con,g}$, $\lambda_{xpd}^{att,g}$, and $\lambda_{mxqd}^{pred,g}$. GCFactors can also be used in the model for the Classes. We will denote a GCFactor effect on the latent classes as $\lambda_{xrd}^{0,g}$, $0 \leq 1 \leq R$, where the superscript 0 refers to the model for the latent classes.

GClasses enter in the conditional logit model for the choices as $\beta^{con,g}_{xm,x^g} + \sum_{p=1}^{P} \beta^{att,g}_{xp,x^g} \cdot z^{att}_{mjitp} + \sum_{q=1}^{Q} \beta^{pred,g}_{xmq,x^g} \cdot z^{pred}_{jitq}$. Inclusion of GClasses in the model for the Classes implies that the $\gamma$ parameters become GClass dependent; that is $\eta_{x|\mathbf{z}_{ji},x^g} = \gamma_{x^g,x0} + \sum_{r=1}^{R} \gamma_{x^g,xr} \cdot z^{cov}_{jir}$. Note that this is similar to a LC Regression analysis, where $x^g$ now plays the role of $x$, and $x$ the role of a nominal $y$ variable.

The remaining linear predictor is the one appearing in the multinomial logistic regression model for the GClasses. It has the form $\eta_{x^g|\mathbf{z}^g_i} = \gamma^g_{x^g,0} + \sum_{r=1}^{R^g} \gamma^g_{x^g,r} \cdot z^{g,cov}_{jr}$. This linear predictor is similar to the one for the Classes (in a standard LC model), showing that GCovariates may be allowed to affect GClasses in the same way that covariates may affect Classes.

Below we will describe the most relevant special cases of this very general latent variable model,[20] most of which were described in Vermunt (2002b, 2003, 2004, and 2005) and Vermunt and Magidson (2005). We then provide some expressions for the exact forms of the various linear predictors in models with GClasses, GCFactors, and GCovariates.

**Model restrictions** One can use the parameter constraints "Class Independent", "No Effect", and "Merge Effects", implying equal $\lambda$'s ($\beta$'s) among all Classes, zero $\lambda$'s ($\beta$'s) in selected Classes, and equal $\lambda$'s ($\beta$'s) in selected Classes.

**ML (PM) estimation and technical settings** Similar to what was discussed in the context of CFactors, with GCFactors, the marginal density $P(\mathbf{y}_j|\mathbf{z}_j)$ described in equation (25) is approximated using Gauss-Hermite quadrature. With three GCFactors and $B$ quadrature nodes per dimension, the approximate density equals

$$P(\mathbf{y}_j|\mathbf{z}_j, \mathbf{z}^g_j) \approx \sum_{x^g=1}^{K^g} \sum_{b_1=1}^{B} \sum_{b_2=1}^{B} \sum_{b_3=1}^{B} P(x^g|\mathbf{z}^g_j)\, P(\mathbf{y}_j|\mathbf{z}_j, x^g, F^g_{b_1}, F^g_{b_2}, F^g_{b_3})\, P^g_{b_1}\, P^g_{b_2}\, P^g_{b_3}.$$

ML (PM) estimates are found by a combination of the upward-downward variant of the EM algorithm developed by Vermunt (2003, 2004) and Newton-

[20] In fact, the multilevel LC model implemented in Latent GOLD Choice is so general that many possibilities remain unexplored as of this date. It is up to Latent GOLD Choice Advanced users to further explore its possibilities.

Raphson with analytic first-order derivatives.[21]

The only new technical setting in multilevel LC Choice models is the same as in models with CFactors; that is, the number of quadrature nodes to be used in the numerical integration concerning the GCFactors. As explained earlier in the context of models with CFactors, the default value is 10, the minimum 2, and the maximum 50.

## 7.2 Application Types

### 7.2.1 Two-level LC Choice model

The original multilevel LC model described by Vermunt (2003) and Vermunt and Magidson (2005b) was meant as a tool for multiple-group LC analysis in situations in which the number of groups is large. The basic idea was to formulate a model in which the latent class distribution (class sizes) is allowed to differ between groups by using a random-effects approach rather than by estimating a separate set of class sizes for each group – as is done in a traditional multiple-group analysis.

When adopting a nonparametric random-effects approach (using GClasses), one obtains the following multilevel LC Choice model:

$$P(\mathbf{y}_j|\mathbf{z}_j) = \sum_{x^g=1}^{K^g} P(x^g) \left[ \prod_{i=1}^{I_j} \sum_{x=1}^{K} P(x|x^g) \prod_{t=1}^{T_i} P(y_{jit}|x, \mathbf{z}_{jit}^{att}, \mathbf{z}_{jit}^{pred}) \right],$$

in which the linear predictor in the logistic model for $P(x|x^g)$ equals $\eta_{x|x^g} = \gamma_{x^g,x0}$. Here, we are in fact assuming that the intercept of the model for the latent classes differs across GClasses.

When adopting a parametric random-effects approach (GCFactors), one obtains

$$P(\mathbf{y}_j|\mathbf{z}_j) = \int_{-\infty}^{\infty} f(F_{1j}^g) \left[ \prod_{i=1}^{I_j} \sum_{x=1}^{K} P(x|F_{1j}^g) \prod_{t=1}^{T} P(y_{jit}|x, \mathbf{z}_{jit}^{att}, \mathbf{z}_{jit}^{pred}) \right] \mathrm{d}F_{1j}^g,$$

where the linear term in the model for $P(x|F_{1j}^g)$ equals $\eta_{x|F_{1j}^g} = \gamma_{x0} + \lambda_{x01}^{0,g} \cdot F_{1j}^g$. Note that this specification is the same as in a random-intercept model for a nominal dependent variable.

[21] Numeric second-order derivatives are computed using the analytical first-order derivatives.

Vermunt (2005) expanded the above parametric approach with covariates and random slopes, yielding a standard random-effects multinomial logistic regression model, but now for a *latent* categorical outcome variable. With covariates and multiple random effects, we obtain

$$P(\mathbf{y}_j|\mathbf{z}_j) = \int_{\mathbf{F}_j^g} f(\mathbf{F}_j^g) \left[ \prod_{i=1}^{I_j} \sum_{x=1}^{K} P(x|\mathbf{z}_{ji}^{cov}, \mathbf{F}_j^g) \prod_{t=1}^{T} P(y_{jit}|x, \mathbf{z}_{jit}^{att}, \mathbf{z}_{jit}^{pred}) \right] \mathrm{d}\mathbf{F}_j^g,$$

where the linear predictor for $x$ equals

$$\eta_{x|\mathbf{z}_{ji},\mathbf{F}_j^g} = \gamma_{x0} + \sum_{r=1}^{R} \gamma_{xr} \cdot z_{jir}^{cov} + \sum_{d=1}^{D^g} \lambda_{x0d}^{0,g} \cdot F_{dj}^g + \sum_{d=1}^{D^g} \sum_{r=1}^{R} \lambda_{xrd}^{0,g} \cdot F_{dj}^g \cdot z_{jir}^{cov}.$$

Also when adopting a nonparametric random-effects approach, one may include covariates in the multilevel LC model; that is,

$$\eta_{x|\mathbf{z}_{ji},x^g} = \gamma_{x^g,x0} + \sum_{r=1}^{R} \gamma_{x^g,xr} \cdot z_{jir}^{cov}.$$

This yields a model for the latent classes in which the intercept and the covariate effects may differ across GClasses. In fact, we have a kind of LC Regression structure in which the latent classes serve as a nominal dependent variable and the GClasses as latent classes.

An important extension of the above nonparametric multilevel LC models is the possibility to regress the GClasses on group-level covariates. This part of the model has the same form as the multinomial logistic regression model for the Classes in a standard LC or FM model.

### 7.2.2 LC discrete-choice models for three-level data

Another application type of the Latent GOLD Choice multilevel option is three-level regression modeling (Vermunt, 2004). A three-level LC conditional logit model would be of the form

$$P(\mathbf{y}_j|\mathbf{z}_j) = \sum_{x^g=1}^{K^g} P(x^g) \left[ \prod_{i=1}^{I_j} \sum_{x=1}^{K} P(x) \prod_{t=1}^{T_i} P(y_{jit}|x, \mathbf{z}_{jit}^{pred}, \mathbf{z}_{jit}^{att}, x^g) \right].$$

Suppose we have a model for first choices with alternative-specific constants and $P$ attributes. The simplest linear predictor in a model that includes

GClasses would then be

$$\eta_{m|\mathbf{z}_{jit},x,x^g} = \beta^{con}_{xm} + \sum_{p=1}^{P} \beta^{att}_{xp}\, z^{att}_{mitp} + \beta^{con,g}_{m,x^g},$$

which is a model in which (only) the constants are affected by the GClasses. A more extended model is obtained by assuming that also the attribute effects vary across GClasses; that is,

$$\eta_{m|\mathbf{z}_{jit},x,x^g} = \beta^{con}_{xm} + \sum_{p=1}^{P} \beta_{xp} \cdot z^{att}_{mitp} + \beta^{con,g}_{m,x^g} + \sum_{p=1}^{P} \beta^{att,g}_{p,x^g} \cdot z^{att}_{mitp}.$$

In practice, it seems to be most natural to allow effects of attributes and predictors that change values across replications to be Class dependent and effects of predictors that change values across cases to depend on the GClasses.

The most extended specification is obtained if all the effects are assumed to be Class dependent, which implies including Classes-GClasses ($x$-$x^g$) interactions. Such a model is defined as

$$\eta_{m|\mathbf{z}_{jit},x,x^g} = \beta^{con}_{xm} + \sum_{p=1}^{P} \beta_{xp} \cdot z^{att}_{mitp} + \beta^{con,g}_{xm,x^g} + \sum_{p=1}^{P} \beta^{att,g}_{xp,x^g} \cdot z^{att}_{mitp}.$$

It should be noted that in each of the above three models, identifying constraints have to be imposed on the parameters involving the GClasses. The attribute effects for the GClasses, for example, are restricted by $\sum_{x^g=1}^{K^g} \beta^{att,g}_{xp,x^g} = 0$, $\beta^{att,g}_{xp,1} = 0$, or $\beta^{att,g}_{xp,K^g} = 0$, for $1 \leq p \leq P$ and $1 \leq x \leq K$. In other words, the parameters in the model for the dependent variable either sum to zero across GClasses, are equal to zero for the first GClass, or are equal to zero for the last GClass.

### 7.2.3 Three-level random-coefficients conditional logit models

Combining the GCFactors from the multilevel model with the CFactors option makes it possible to specify three-level random-coefficient conditional logit models. These are similar to other types of three-level GLM regression models with parametric random effects (Im and Gionala, 1988; Skrondal and Rabe-Hesketh, 2004; Rodriguez and Goldman, 2001; Vermunt, 2002c, 2004). In terms of probability structure, this yields:

$$P(\mathbf{y}_j|\mathbf{z}_j) = \int_{\mathbf{F}^g_j} f(\mathbf{F}^g_j) \left[ \prod_{i=1}^{I_j} \int_{\mathbf{F}_{ji}} f(\mathbf{F}_{ji}) \prod_{t=1}^{T_i} P(y_{jit}|\mathbf{z}^{pred}_{jit}, \mathbf{z}^{att}_{jit}, \mathbf{F}_{ji}, \mathbf{F}^g_j) \mathrm{d}\mathbf{F}_{ji} \right] \mathrm{d}\mathbf{F}^g_j.$$

The simplest special case is obtained by assuming that the conditional logit model contains random intercepts at both the case and the group level. The corresponding linear predictor in a model with $P$ attributes equals

$$\eta_{m|\mathbf{z}_{jit},F_{1ji},F^g_{1j}} = \beta^{con}_m + \sum_{p=1}^{P} \beta_{xp} \cdot z^{att}_{mjitp} + \lambda^{con}_{m1} \cdot F_{1ji} + \lambda^{con,g}_{m1} \cdot F^g_{1j}.$$

Such a model containing a single CFactor and a single GCFactor will suffice in most three-level random-effects applications. However, similar to the random effects models discussed in the context of the CFactors option, this model can be expanded with random slopes at both levels using the factor-analytic or generalized random-effects specification illustrated in equation (24).

### 7.2.4 LC growth models for multiple response

Suppose one has a longitudinal data set containing multiple responses for each time point. The multiple responses could be used to build a time-specific latent classification, while the pattern of (latent) change over time could be described using a (LC) growth model. Specification of such a model would involve using the index $i$ for the time points and the index $j$ for the cases (time points are nested within cases). The LC model for the time points would be a LC Choice model. The multinomial logistic regression model for the (time-specific) latent classes will have the form of a LC growth model: class membership depends on time, where the intercept and possibly also the time slope is allowed to vary across individuals. This variation can be modelled using continuous random effects (GCFactors) and/or discrete random effects (GClasses).

### 7.2.5 Two-step IRT applications

Another application of the Latent GOLD Choice multilevel option is in IRT models for educational testing that assume a two-stage response process (Bechger et al., 2005; Westers and Kelderman, 1991, 1993). These models associate a discrete (usually binary) latent response to each observed item response, where a standard IRT model is specified for the discrete latent responses. A specific mechanism is assumed for the relationships between the latent and observed item responses. In Westers and Kelderman's SERE model, for example, the first latent class knows and the second latent class does not know the correct answer on a multiple choice item, implying that

the first class gives the correct answer with probability one and the other class guesses with probabilities that depend on the attractiveness of the alternatives. Using the Latent GOLD Choice notation, the SERE model would be defined as a Rasch-like model for the latent classes

$$\eta_{x|\mathbf{z}_i,F_j^g} = \gamma_{x0} + \sum_{r=1}^{R} \gamma_{xr} \cdot z_{jir}^{cov} + \lambda_x^{0,g} \cdot F_j^g,$$

where the covariates are item dummies. The constraint that $P(y_{ji} = 1|x = 1) = 1.00$ can be imposed by using the offset option, and $P(y_{ji} = m|x = 2)$ is left unrestricted.

### 7.2.6 Non multilevel models

The final use of the multilevel option we describe here does not yield a multilevel model, but is a trick for estimating models that cannot be estimated any other way. The trick consists of using a Group ID variable that is identical to the Case ID or, equivalently, to have groups that consist of no more than one case each. GCFactors can then be used as CFactors. This makes it possible to define models in which CFactors affect the latent classes. Another possibility is to use the GClasses as an additional case-level nominal latent variable, yielding a model in which one nominal latent variable may affect another nominal latent variable.

# 8 Complex Survey Sampling

The Survey option makes it possible to obtain consistent parameter estimates and correct standard errors with complex sampling designs. This option can be used in combination with any model that can be estimated with Latent GOLD Choice. Parameter estimation is based on the so-called pseudo-ML estimator that uses the sampling weights as if they were case weights. Correct statistical tests with stratified and clustered samples, as well as with sampling weights and samples from finite populations are obtained using the linearization variance estimator.

Latent GOLD Choice also implement an alternative method to deal with sampling weights. This is a two-step procedure in which the model is first estimated without making use of the sampling weights, and in which subsequently the latent class sizes and covariate effects are corrected using the sampling weights.

## 8.1 Pseudo-ML Estimation and Linearization Estimator

The survey option can be used to take into account the fact that cases may

1. belong to the same stratum,
2. belong to the same primary sampling unit (PSU), often referred to as a sampling cluster,
3. contain a sampling weight,
4. be sampled from a finite population.

Let $o$ denote a particular stratum, $c$ a particular PSU in stratum $o$, and $i$ a particular case in PSU $c$ of stratum $o$. Moreover, let $O$ be the number of strata, $C_o$ the number of PSUs in stratum $o$, and $I_{oc}$ the number of cases in PSU $c$ of stratum $o$. The sampling weight corresponding to case $i$ belonging to PSU $c$ of stratum $o$ is denoted by $sw_{oci}$, and the population size (total number of PSUs) of stratum $o$ by $N_o$.[22]

From this notation, it can be seen that PSUs are nested within strata, and that cases are nested within PSUs. In other words, records with the same Case ID should belong to the same PSU, and all records with the same PSU identifier should belong to the same stratum. The population size $N_o$ indicates the population number of PSUs in stratum $o$, and should thus have the same value across records belonging to the same stratum. Another thing that should be noted is that in *multilevel models*, the strata, PSUs, and sampling weights concern groups rather than cases; that is, one has strata and PSUs formed by groups and sampling weights for groups.

For parameter estimation, only the sampling weights need to be taken into account. When sampling weights are specified, Latent GOLD Choice will estimate the model parameters by means of pseudo-ML (PM) estimation (Skinner, Holt, and Smith, 1989). Recall that ML estimation involves maximizing

$$\log \mathcal{L} = \sum_{i=1}^{I} w_i \log P(\mathbf{y}_i|\mathbf{z}_i, \boldsymbol{\vartheta}),$$

[22] In Latent GOLD Choice, one can either specify the fraction $\frac{C_o}{N_o}$ or the population size $N_o$. If the specified number in "Population Size" is smaller than 1 it is interpreted as a fraction, otherwise as a population size.

where $w_i$ is a case weight. In pseudo-ML estimation, one maximizes

$$\log \mathcal{L}_{pseudo} = \sum_{o=1}^{O} \sum_{c=1}^{C_c} \sum_{i=1}^{I_{oc}} sw_{oci} \log P(\mathbf{y}_{oic}|\mathbf{z}_{oic}, \boldsymbol{\vartheta}),$$

which is equivalent to maximizing $\log \mathcal{L}$ using the sampling weights as if they were case weights. In Latent GOLD Choice one may also have both case and sampling weights, in which case we get

$$\log \mathcal{L}_{pseudo} = \sum_{o=1}^{O} \sum_{c=1}^{C_c} \sum_{i=1}^{I_{oc}} w_{oci} \cdot \left[ sw_{oci} \log P(\mathbf{y}_{oic}|\mathbf{z}_{oic}, \boldsymbol{\vartheta}) \right],$$

which is equivalent to performing ML estimation using the $sw_{oci} \cdot w_{oci}$ as "case" weights.

Each of the four complex sampling characteristics is taken into account by the so-called linearization estimator of variance-covariance matrix of the parameter estimates (Skinner, Holt, and Smith, 1989). Application of this method in the context of FM and LC models was proposed by Vermunt (2002b) and Wedel, Ter Hofstede, and Steenkamp (1998). The overall structure of $\widehat{\Sigma}_{survey}(\boldsymbol{\vartheta})$ is similar to the robust or sandwich estimator $\widehat{\Sigma}_{robust}(\boldsymbol{\vartheta})$ discussed earlier; that is,

$$\widehat{\Sigma}_{survey}(\boldsymbol{\vartheta}) = \widehat{\mathbf{H}}^{-1} \widehat{\mathbf{B}} \ \widehat{\mathbf{H}}^{-1}.$$

As can be seen, a matrix $\mathbf{B}$ is "sandwiched" between the inverse of the Hessian matrix. For the computation of $\mathbf{B}$, one needs two components: the contribution of PSU $c$ in stratum $o$ to the gradient of parameter $k$, denoted by $g_{ock}$, and its sample mean in stratum $o$, denoted by $\overline{g}_{ok}$. These are obtained as follows:

$$g_{ock} = \sum_{i=1}^{I_{oc}} sw_{oci} \frac{\partial \log P(\mathbf{y}_{oci}|\mathbf{z}_{oci}, \boldsymbol{\vartheta})}{\partial \vartheta_k}$$

and

$$\overline{g}_{ok} = \frac{\sum_{c=1}^{C_o} g_{ock}}{C_o}$$

Using these two components, element $B_{kk'}$ of $\mathbf{B}$ can be defined as

$$B_{kk'} = \sum_{o=1}^{O} \frac{C_o}{C_o - 1} \left(1 - \frac{C_o}{N_o}\right) \sum_{c=1}^{C_o} (g_{ock} - \overline{g}_{ok})(g_{ock'} - \overline{g}_{ok'}).$$

Note that if we neglect the finite population correction factor $(1-\frac{C_o}{N_o})$, $\mathbf{B}$ is the sample covariance matrix of the PSU-specific contributions to the gradient vector.

Various observations can be made from the formula for $B_{kk'}$. The first is that without complex sampling features (one stratum, single case per PSU, no sampling weights, and $\frac{C_o}{N_o} \approx 0$), the above procedure yields $\widehat{\boldsymbol{\Sigma}}_{robust}(\boldsymbol{\vartheta})$, which shows that $\widehat{\boldsymbol{\Sigma}}_{survey}(\boldsymbol{\vartheta})$ not only takes into account the sampling design, but is also a robust estimator of $\boldsymbol{\Sigma}(\boldsymbol{\vartheta})$. Second, the fact that gradient contributions are aggregated for cases belonging to the same PSU shows that the PSUs are treated as the independent observational units, which is exactly what we want. Third, the term $\frac{C_o}{C_o-1}$ is only defined if each stratum contains at least two PSUs: Latent GOLD Choice "solves" this problem by skipping strata for which $C_o = 1$ and by giving a warning that this happens. A common solution to this problem is to merge strata.

The design effect corresponding to a single parameter equals the ratio of its design corrected variance and its variance assuming simple random sampling. A multivariate generalization is obtained as follows (Skinner, Holt, and Smith, 1989):

$$\begin{aligned} deff &= \text{tr}\left[\widehat{\boldsymbol{\Sigma}}_{standard}(\boldsymbol{\vartheta})^{-1}\widehat{\boldsymbol{\Sigma}}_{survey}(\boldsymbol{\vartheta})\right] / npar \\ &= \text{tr}\left[(-\widehat{\mathbf{H}})\,\widehat{\mathbf{H}}^{-1}\,\widehat{\mathbf{B}}\;\widehat{\mathbf{H}}^{-1}\right] / npar = \text{tr}\left[-\widehat{\mathbf{B}}\;\widehat{\mathbf{H}}^{-1}\right] / npar, \end{aligned}$$

where "tr" is the trace operator. The generalized design effect is thus the average of the diagonal elements of $-\widehat{\mathbf{B}}\,\widehat{\mathbf{H}}^{-1}$. Note that this number equals the average of the eigenvalues of this matrix.

## 8.2 A Two-step Method

Latent GOLD Choice also implements an alternative two-step method for dealing with sampling weights in LC analysis, which was described in Vermunt (2002b) and Vermunt and Magidson (2001). The procedure involves performing an unweighted analysis followed by a weighted analysis in which the parameters in the model part for the response variables are fixed to their unweighted ML (PM) estimates. More specifically, in step two, the class sizes and the covariates effects are adjusted for the sampling weights. The adjusted log-likelihood function that is maximized equals

$$\log \mathcal{L}_{adjusted} = \sum_{i=1}^{I} sw_i \log \sum_{x=1}^{K} P(x|\mathbf{z}_i, \boldsymbol{\vartheta}_{adjusted})\, P(\mathbf{y}_i|x, \mathbf{z}_i, \widehat{\boldsymbol{\vartheta}}_{ML}),$$

where $\boldsymbol{\vartheta}_{adjusted}$ are the unknown parameters to be estimated.

The rationale of this procedure is that an unweighted analysis may yield more stable (more efficient) estimates for the parameters defining the latent classes, but yields biased class sizes and covariate effects. The latter are corrected in the second step of the procedure.

# 9 Latent Gold Choice's Advanced Output

This section describes the changes and additional items in the Latent GOLD Choice output sections when the Advanced options are used.

## 9.1 Model Summary

For multilevel models, the first part of the Model Summary output reports the number of groups ($J$) in addition to the number cases and replications. When the Survey option is used, the program reports the generalized design effect ($deff$), which is an overall measure indicating how many times larger the design corrected variances are compared to the asymptotic variances.

For multilevel models, *Chi-squared Statistics* are not reported and the bootstrap $L^2$ and $-2LL$-difference options are not available. When the Survey option is used, the bootstrap-based $L^2$ and $LL$-difference tests are corrected for the complex sampling design by multiplying the bootstrap replications' $L^2$ and $-2LL$-difference values by the generalized design effect $deff$. Note that the bootstrap replication samples themselves are obtained by simple random sampling.

In multilevel models, as in all other Latent GOLD Choice models, the number of cases serves as $N$ (sample size) in the computation of the BIC and CAIC values that appear in the *Log-likelihood Statistics*. An alternative would have been to assume $N$ to be equal to the number of groups instead of the number of cases. Users who prefer this alternative definition of BIC and CAIC may compute these statistics themselves.

The *Classification Statistics* contain information on how well one can predict an individual's CFactor scores and a group's GClass membership and GCFactor scores. For GClasses, one obtains the same information as for the latent classes (proportion of classification errors and three $R^2$ measures). For CFactors and GCFactors, one obtains only the standard $R^2$, which can

be interpreted as a reliability measure. In multilevel models with covariates, *Covariate Classification Statistics* will contain information for the GClasses.

The *Prediction Statistics* are the same as in models without CFactors, GClasses, and GCFactors. The $R_y^2$ measures indicates how well a model predicts the choices given all predictors, covariates, and latent variables.

## 9.2 Parameters

This section reports the parameters corresponding to CFactors, GClasses, and GCFactors. CFactors, GClasses, and GCFactors effects may appear in the *Model for Choices/Rankings/Ratings*. In multilevel models, GClasses and GCFactors may be used in the *Model for Classes*. When GClasses affect a particular term (the intercept or a covariate effect), one obtains a separate set of coefficients for each GClass. GCFactors enter as random effects in the regression model for the discrete latent variable(s). In models with GClasses, the parameters output contains the coefficients of the multinomial logistic regression *Model for GClasses*.

The reported Class-specific $R_{y|x}^2$ measures are obtained by averaging the predicted values over the other latent variables included in the model. This is the reason that in a one-Class model, the "Class-specific" $R_{y|1}^2$ may be lower that the overall $R_y^2$.

When the Survey option is used, one obtains design corrected standard errors and Wald statistics.

In models with CFactors, one obtains an output subsection called *Random Effects*. This subsection provides the CFactor effects $\mathbf{\Lambda}$ and the variance-covariance matrix of the random effects, $\mathbf{\Sigma_\Psi} = \mathbf{\Lambda}\,\mathbf{\Lambda}'$.

## 9.3 GProfile

The first part of this output section reports the sizes of the GClasses $[P(x^g)]$ and the probability of being in a certain latent class for each GClass $[P(x|x^g)]$. The second part of the GProfile section reports the GClass-specific probabilities for the choice variable. The computation of this part of the GProfile output is similar to the computation of the same kinds of numbers in the Profile output.

## 9.4 ProbMeans

In models with CFactors, the Probmeans output reports the average CFactor posterior mean for each covariate category.

## 9.5 Frequencies

Frequencies are not reported in multilevel LC models.

## 9.6 Classification

The *Standard Classification* output provides information on the CFactor and GCFactor posterior means $\widehat{E}(F_{di}|\mathbf{z}_i, \mathbf{y}_i)$ and $\widehat{E}(F^g_{dj}|\mathbf{z}_j, \mathbf{y}_j)$, the GClass posterior probabilities $\widehat{P}(x^g|\mathbf{z}_j, \mathbf{y}_j)$, and the modal GClass for each data pattern. The posterior means are obtained using Gauss-Hermite quadrature; for example,

$$\begin{aligned}\widehat{E}(F_{di}|\mathbf{z}_i, \mathbf{y}_i) &= \frac{\int_{-\infty}^{\infty} F_{di}\ P(\mathbf{y}_i|\mathbf{z}_i, F_{di})\,\mathrm{d}\,F_{di}}{\int_{-\infty}^{\infty} P(\mathbf{y}_i|\mathbf{z}_i, F_{di})\,\mathrm{d}\,F_{di}} \\ &\approx \frac{\sum_{b_d=1}^{B} F_{b_d}\ P(\mathbf{y}_i|\mathbf{z}_i, F_{b_d})\,P_{b_d}}{\sum_{b_d=1}^{B} P(\mathbf{y}_i|\mathbf{z}_i, F_{b_d})\,P_{b_d}}.\end{aligned}$$

In multilevel models with covariates, the *Covariate Classification* output section reports the GClass membership probabilities given group-level covariates $[\widehat{P}(x^g|\mathbf{z}^g_j)]$.

## 9.7 Output-to-file Options

The *Standard Classification* option can be used to write the CFactors and GCFactors posterior means, the GClasses posterior probabilities, and the modal GClass to an output file. In models with GClasses, *Covariate Classification* saves the classification of groups into GClasses based on group covariates to the output file.

The *Individual Coefficients* corresponding to CFactor effects are computed in a special way:

$$\widehat{\lambda}_{iqd} = \sum_{x=1}^{K} \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i)\ \left[\widehat{E}(F_{di}|\mathbf{z}_i, \mathbf{y}_i, x)\,\widehat{\lambda}_{xqd}\right],$$

where $\widehat{E}(F_{di}|\mathbf{z}_i, \mathbf{y}_i, x)$ is the posterior mean of $F_{di}$ given that $i$ belongs to latent class $x$. The $\widehat{\lambda}_{iqd}$ can be used together with the $\widehat{\beta}_{iq}$ to obtain HB-like predicted values for case $i$. The posterior standard deviation of $\widehat{\lambda}_{iqd}$ equals

$$\widehat{\sigma}_{\widehat{\lambda}_{iqd}} = \sqrt{\sum_{x=1}^{K} \widehat{P}(x|\mathbf{z}_i, \mathbf{y}_i) \left[\widehat{E}(F_{di}|\mathbf{z}_i, \mathbf{y}_i, x)\, \widehat{\lambda}_{xqd} - \widehat{\lambda}_{iqd}\right]^2},$$

HB-like individual coefficients for a "full" intercept or predictor term may also be obtained by summing the various individual coefficient components for that term. For example, for a random-intercept model such as given in equation (23), the HB-like individual coefficient for a "full" alternative-specific constant is computed by summing $\widehat{\beta}_{im}^{con}$ and $\widehat{\lambda}_{im1}^{con}$.

In multilevel models, the *Cook's D* value is computed per group rather than per case. Thus, rather than for detecting influential cases, it can be used for detecting influential groups.

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# 11 Notation

## 11.1 Basic Models

| | |
|---|---|
| $P(\cdot)$ | probability |
| $i$, $I$ | case index, # of cases |
| $t, T_i$ | replication index, # of replications for case $i$ |
| $y_{it}$ | response of case $i$ at replication $t$ |
| $m$ | category of the response variable |
| $y_m^*$, | score assigned to category $m$ of a rating variable |
| $x$ | nominal latent variable, a particular latent class |
| $K$ | # of latent classes |
| $r$, $R$ | covariate index, # of covariates |
| $p$, $P$ | attribute index, # of attributes |
| $q$, $Q$ | predictor index, # of predictors |
| $z_{ir}^{cov}$ | covariate |
| $z_{mitq}^{att}$ | attribute |
| $z_{itq}^{pred}$ | predictor |
| $\eta$ | linear predictor |
| $\beta$, $\gamma$ | parameter in model for $y_{it}$, parameter in model for $x$ |
| $\tau_{ix}$ | known-class indicator |
| $w_i$ | case weight |
| $v_{it}$ | replications weight |
| $u$, $U$ | "covariate" pattern index, # of "covariate" patterns |
| $i^*$, $I^*$ | unique data pattern index, # of unique data patterns |
| $N$ | total sample size (after weighting) |

## 11.2 Advanced Models

| | |
|---|---|
| $d$, $D$ | CFactor index, # of CFactors |
| $F_{id}$ | scores of case $i$ CFactor $d$ |
| $\lambda_d$ | an effect of CFactor $d$ |
| $j$, $J$ | group index, # of groups |
| $I_j$ | # of cases in group $j$ |
| $y_{jit}$ | response of case $i$ of group $j$ at replication $t$ |
| $\mathbf{y}_j$ | vector of responses of group $j$ |
| $g$ | group-level quantity |
| $x^g$ | group-level nominal latent variable, a particular group GClass |
| $z_{jr}^{g,cov}$ | group-level covariate |
| $F_{jd}^g$ | score of group $j$ of group-level continuous factor (GCFactor) $d$ |
| $\gamma^g, \beta^g, \lambda^g$ | group-level parameters |
| $o$, $O$ | stratum, # of strata |
| $c$, $C_o$ | PSU, # of PSU's in stratum $o$ |
| $sw_{oci}$ | sampling weight |
| $I_{oc}$ | # of cases in PSU $c$ of stratum $o$ |
| $N_o$ | total # of PSUs in population in stratum $o$ |

# Part 3: Using Latent GOLD Choice

## 1.0 Overview

This part of this manual describes and illustrates the use of the program. Section 2 contains a general description of all the features and menus that are available for the estimation of discrete choice models. Section 3 presents a sample tutorial to get you up and running quickly, "Analyzing data from a simulated choice experiment using the 3-file format", that describes the basic usage of the program.. Other tutorials, available as separate files, include the following:

Choice_tutorial1A.pdf, Tutorial #1A: Using the SI-CHAID add-on to profile latent class segments

**Note: Tutorials 2-7 are currently being updated for version 4.0:**

Choice_tutorial2.pdf. Tutorial #2: Using the Results of the Model to Predict / Simulate Future Choices.
Choice_tutorial3.pdf: Tutorial #3: Estimating Brand and Price effects.
Choice_tutorial4.pdf: Tutorial #4: Using the 1-file Format.
Choice_tutorial5.pdf: Tutorial #5: Analyzing Ranking data.
Choice_tutorial6.pdf, Tutorial #6: Using LG Choice to Estimate max-diff (best-worst) and Other Partial Ranking Models
Choice_tutorial7.pdf, Tutorial #7: LC Segmentation with Ratings-based Conjoint Data
Choice_tutorial7-2.pdf, Tutorial #7A: LC Segmentation with Ratings-based Conjoint Data

All of these tutorials are also available online at
http://www.statisticalinnovations.com/products/choice.html#tutorialslink
along with other tutorials that are under development. For example, Tutorial #8 illustrates how to use the program to estimate allocation (constant sum) models. For more information on allocation, see Section 2.5 of the Technical Guide.

## 2.0 Introduction

Using Latent GOLD Choice is easy. Typically, using Latent GOLD Choice to estimate a new model involves the following steps:

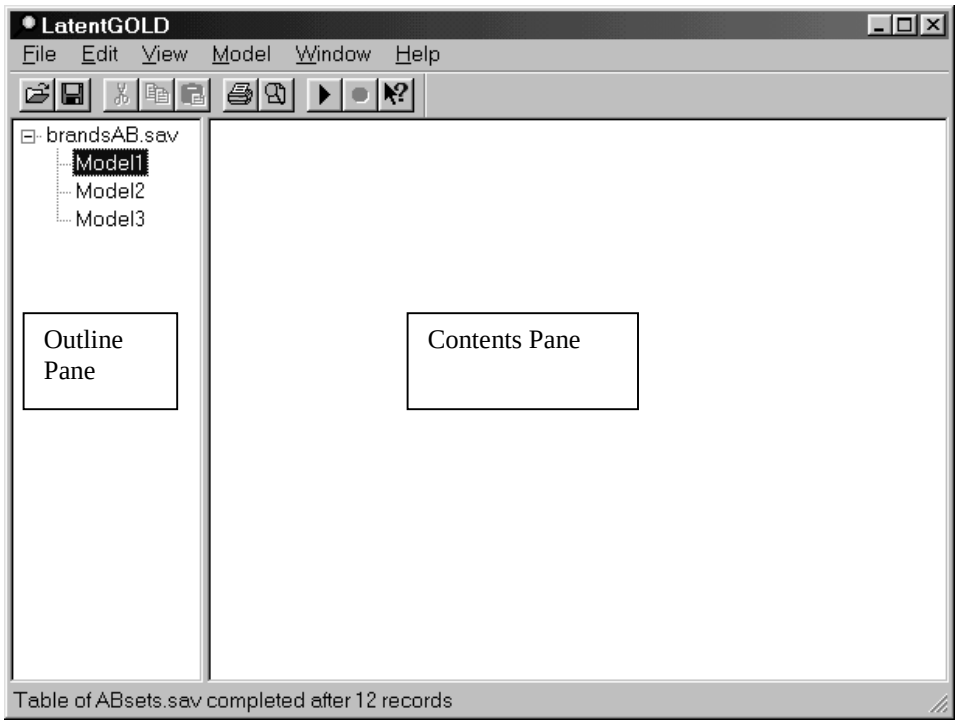
Model Setup
Specifying Output Options
Model Estimation
Viewing Output

## Step 1: Model Setup

A typical session with Latent GOLD Choice analysis begins by opening your data file(s) and selecting your model settings. Your data file(s) may be SPSS .sav files, ASCII text (.txt, .dat) files, a Sawtooth Software .cho file, or if you have the optional DBMS/Copy interface, SAS, Excel and several other file formats. While you may prefer to use the single data file format, Latent GOLD Choice also allows a more flexible 3-file format. When using the 3-file format, you begin by opening the response file that contain the choices to each of the choice sets.

Alternatively, you may begin by retrieving previously used setups. Opening a previously saved .lgf file will retrieve the setups for one or more models that have previously been saved. Examples of the use of the FILE OPEN to open a response file and to open an .lgf file are given in tutorial #1. Upon opening any file, your initial screen consists of two parts - the Contents Pane and the Outline Pane:

**Figure 1. Outline Pane and Contents Pane**




Names for each of the models appear in the left-hand, or Outline Pane. Once a model is estimated, it's results appear in the right-hand, or Contents Pane. The Outline Pane is used to select different model views and summaries, while the Contents Pane contains the actual views and summary output.

The 3-file format consists of:
an *alternatives file* which defines each alternative in terms of one or more *attributes*,
a *choice sets file* containing one or more *choice sets*, each presenting a set of alternatives among which the choice(s) are made.
a *response file* indicating the choices made by each respondent to one or more *choice sets*.

**Response file** (required). The response file contains the choices made by each respondent to one or more *choice sets* defined in an associated *sets file*. Optionally, the response file may also contain *respondent characteristics* (age, gender) or choice-set characteristics, for inclusion in the model as covariates or predictors. Generally, the Response file will also contain a case ID and sets ID variables. In the special case that no attributes are included in the model, alternative-specific constants (alphas)[1] must be included to estimate a minimal model (which may also contain predictors and/or covariates). In this special case of no attributes, for a dependent defined as "choice", this minimal model reduces to the traditional latent class MNL model and the results of estimation will be equivalent to a corresponding model estimated using the Latent GOLD Regression module where the dependent variable scale type is set to 'Nominal'. For a rating, this yield a model equivalent to an ordinal regression model.

[1] Alternative-specific constants are entered into the model by double-clicking on the special variable named '_Constants_' that shows up automatically in the Attributes tab.

When the 3-file format is used, a response file is always required. The Alternatives and Choice set files may be omitted if only the alternative-specific constants ('_Constants_') are included in the model. Otherwise, effects for attributes defined in the Alternatives file may be included in the model in addition to or in place of the alternative-specific-constants. After opening the Response File, the remaining files are opened in the Attributes Tab, where the attributes effects to be included in the model are defined.

**Several examples of Alternatives and Sets files are given in Tutorials #1 and #2. More specific information is given below.**

**Alternatives file**. Each record (row) in this file defines a distinct choice *alternative* in terms of one or more *attributes*. A unique label identifies each of these alternatives. In figure 1 below, the unique identification label is named 'alt_id'. The other 3 variables shown correspond to *attributes* describing the various *alternatives* -- BRAND, PRICE, and NOBUY. Note that the value of the attributes BRAND and PRICE are specified as missing (".") for the NONE alternative. In the case of a non-numeric (string, character) attribute, the character(s) used to denote a missing value (" . " in the example below) needs to be formally defined as a missing value in an SPSS .sav file. For ASCII files, the character sequence 'open quote followed by a closed quote' (""), is recognized as missing by Latent GOLD Choice.

Attributes for which main effects are estimated are specified in the Attributes tab by selecting them from the variable list, moving them to the Attributes box and setting the desired scale types (see Tutorial #1). Various interaction effects can also be included in the model by defining appropriate interaction variables and selecting them in addition to the included main effects. Tutorial #2 shows how to include BRAND by PRICE interactions in the model by constructing the interaction variables PRICE.A, PRICE.B.

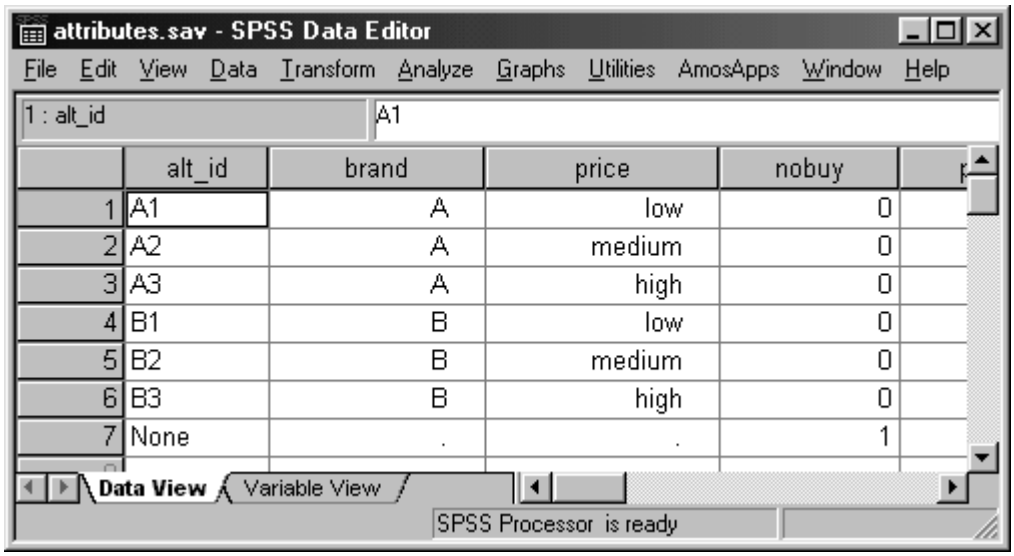

attributes.sav - SPSS Data Editor
File Edit View Data Transform Analyze Graphs Utilities AmosApps Window Help
1 : alt_id
A1
alt_id brand price nobuy
1 A1 A low 0
2 A2 A medium 0
3 A3 A high 0
4 B1 B low 0
5 B2 B medium 0
6 B3 B high 0
7 None . . 1
Data View
Variable View
SPSS Processor is ready

**Figure 2: Example of an SPSS formatted *alternatives file* defining 7 alternatives.**

**Sets file.** Each record in this file defines a choice set by listing the corresponding identification labels for each choice *alternative* included in the set.

format: setid# 1st alternative id# 2nd alternative id# …

where setid# is the set # (generally, a sequential index number 1,2,3…), and the jth alternative id# corresponds to a valid alternative identification number given in the alternatives file. It should be noted that choice sets may have different numbers of alternatives, in which case some of the alternatives need to be defined as missing. Moreover, the sets file may also contain sets that are not presented to the respondents

(inactive sets), for which one will also obtain estimates of the class-specific and overall choice probabilities.

After you have opened your Response File, you are ready to specify the model settings in the Variables Tab. Bring up the Model Analysis Dialog Box by double-clicking on the model name in the Outline Pane. Then, set up your model by specifying the name of the dependent variable which contains the respondent choices, the case ID, set ID, any covariates, the attribute effects to be included in the model, and any technical settings. This process is illustrated in tutorial #1.

## Model Analysis Dialog Box

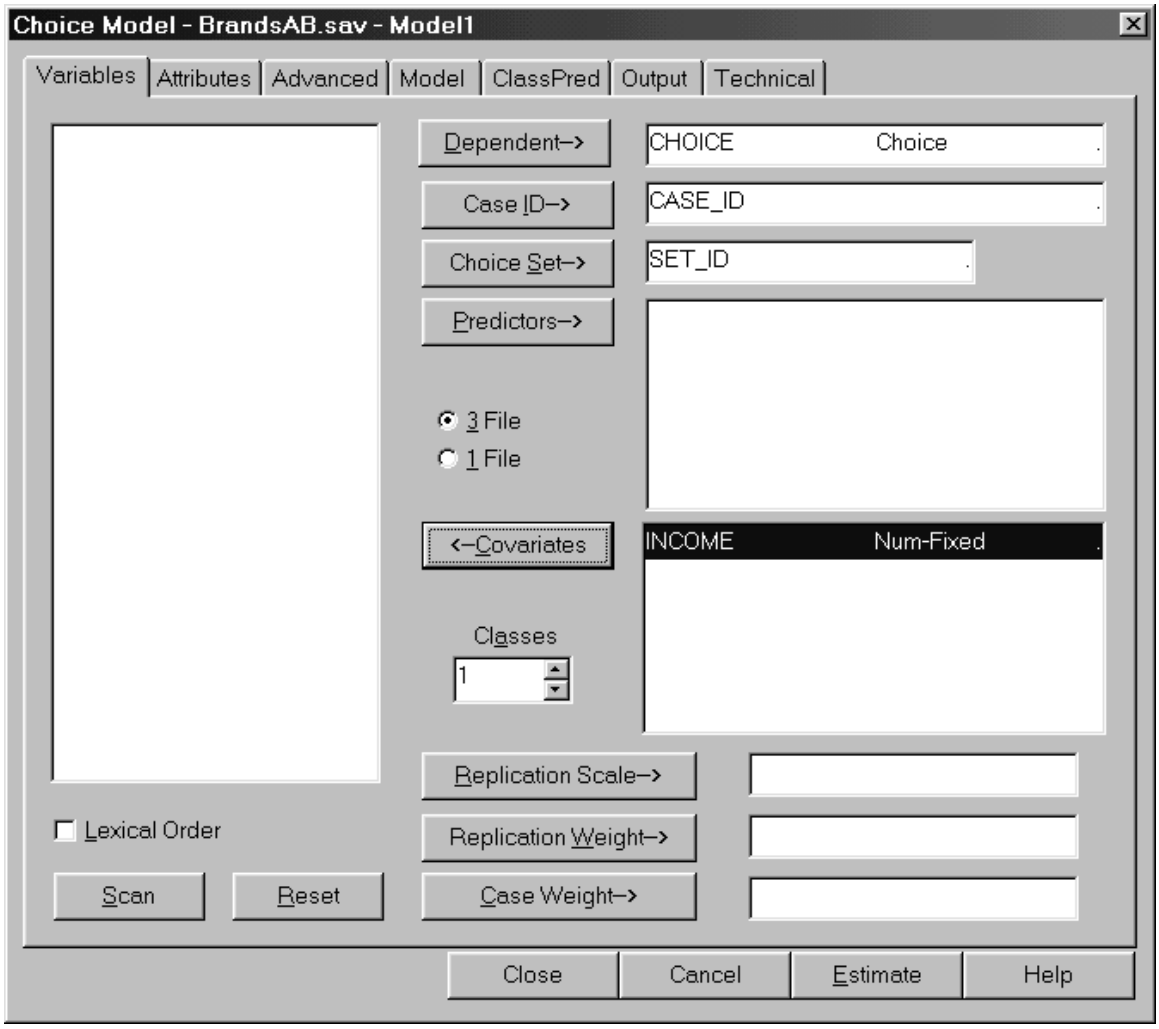



**Figure 3. Model Analysis Dialog Box**

The Model Analysis Dialog Box contains several tabs that are visible on top. The tabs are:

Variables Tab
Attributes Tab
Advanced Tab
Model Tab
ClassPred Tab

Output Tab
Technical Tab

By default, the Model Analysis Dialog Box opens to the Variables Tab.

## Variables Tab

File Format Radial Button -- **3 File** vs. **1 File**. By default, a 3-file format is assumed. To change the setting, click the circle to the left of the appropriate file format. If the 1-file format is selected, the attributes are assumed to be included on the file. The Attributes Tab list all variables on the file and the Alternatives button is deactivated. For choice and ranking models, the 1-file format contains 1 record per alternative, whereas the 3-file format contains 1 record per set. For rating models, there is one record per set in both file format.

All variables that may be included in the analysis are listed in the "Variables" box. Variables may be designated as one of these types: *Dependent Variable, Case ID, Choice Set, Predictors, Covariates, Replication Scale, Replication Weight, Case Weight Variable*. A dependent variable must be specified in order to begin an analysis. To select a variable, highlight the variable name, then click on the appropriate arrow key to move the variable into the corresponding box.
**Dependent Variable**: assign one variable to be used as the dependent variable

**Dependent Variable Types**
The Dependent Variable may specify a single choice within each set, 2 or more choices (ranking and partial ranking models) or a rating. By default the scale type is set to Choice. To set the scale type for the dependent variable, right click on the variable in the Dependent Variable box and the following menu pops up, listing the different **Dependent Variable Types:**

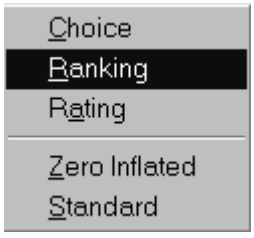



**Figure 4. Dependent Variable Types Menu**

In the 3-file format, the value of the dependent variable equals between 1 and the number of alternatives in the choice set concerned, In ranking models, the order of the records (with the same choice set ID) in the response file indicates the rank of the alternatives,

In the 1-file format,
- for choice models, the dependent variable equals 1 for the selected alternative and 0 otherwise
- for ranking models, it equals 1, 2, 3, etc. for the selected alternatives, where the number indicates the rank, and 0 for the non-selected alternatives in the case of a partial ranking
- for rating models, it equals the rating (as in the 3-file format)

**Case ID**: assign one variable to be used as Case ID. If *no* Case ID variable is specified, the program assumes that the data file contains only a single choice set – that is, only 1 record (in the 3-file format) or 1set of alternatives records (in the 1-file format) per case. For files in which one or more cases have multiple choice sets, assign one variable as a Case ID variable that uniquely identifies a “case”.

**Choice Set**: assign one variable to be used to identify each Choice Set. The choice set variable may be numeric or character. In the 3-file format, this variable is used to link the response file to the set file, whereas in the 1-file format it indicates which alternatives records belong to the same set.

**Predictors**: assign one or more variables to be used as predictors. Predictors may be either Numeric or Nominal. By default, variables containing values are treated as Numeric (labeled "Num-Fixed") and character variables are treated as Nominal. To change the scale type, right click on a predictor in the predictor box and select either Numeric or Nominal. Predictors are explanatory variables that are constant across alternatives, but may vary across sets.

**Covariates**: assign one or more variables to be used as covariates. Covariates represent variables that are *descriptive* or *predictive* of the latent variable, not of the dependent variable. Use of these variables is desirable for describing differences between classes and in reducing classification error.

**Covariate Types**
Right clicking on any variable included in the covariate box and the following menu pops up, listing the different **Covariate Types**

**Figure 5. Covariate Types Menu**

**Numeric -** set the covariate(s) to numeric
**Nominal -** set the covariate(s) to nominal
**Active –** the default setting
**Inactive** - set covariates to be inactive
**Group** - set covariates to apply to group-level (Gclasses)

Active covariates will generally effect the definition of the classes and the part-worth and other parameters. Advantages of working with inactive instead of active covariates are that the estimation time is not increased and that the obtained solution is the same as without covariates (i.e., inactive covariates do not influence the parameter estimates).

The box labeled **Classes** is located beneath the Covariates button. Enter a number greater than 0. Separate models will be estimated for each segment. If a range is specified for the number of classes, such as '1-4', separate sets of models will be estimated, the first representing a 1-class model (the traditional aggregate model which assumes a single homogeneous population).

**Replication Scale**: assign one variable to be used as Replication Scale

**Replication Weight**: assign one variable to be used as Replication Weight

**Case Weight Variable**: assign one variable to be used as Case Weight

**SCAN**: Scans the data file(s) to obtain all values and labels for the variables.

Once you have finished in the Variables Tab, you may wish to click the Scan button to check to see that the variables are being read correctly. After scanning the file, you may double click on the Dependent or any Predictor or Covariate to view the categories / values.

**GROUPS**. Groups adjacent categories / values for the Dependent or any Predictor or Covariate to reduce the total number of levels. For numeric variables, the value for a group is the mean of the values within that group. To implement the grouping, double click on a variable to open the SCORE window. At the bottom right next to the GROUPS label enter a number for the desired number of groups and click the Group button. The number of groups will become the desired number or fewer groups.

**USER SCORES /Uniform Scores**. Re-codes the scores assigned to numeric variables to other values. To implement, double click on a variable to open the SCORE window. Double click on any category / value for that variable. In the bottom left box, replace the current score by the desired new score and click the Replace button. To change scores to equi-distant integers, click the Uniform button. Click OK when finished.

After completing the variable assignments in the Variables tab, open the Attributes tab and continue the model setup. In the Attributes tab you can Open Alternatives and Sets Files, and specify the attributes and scale types for these attributes.

## Attributes Tab

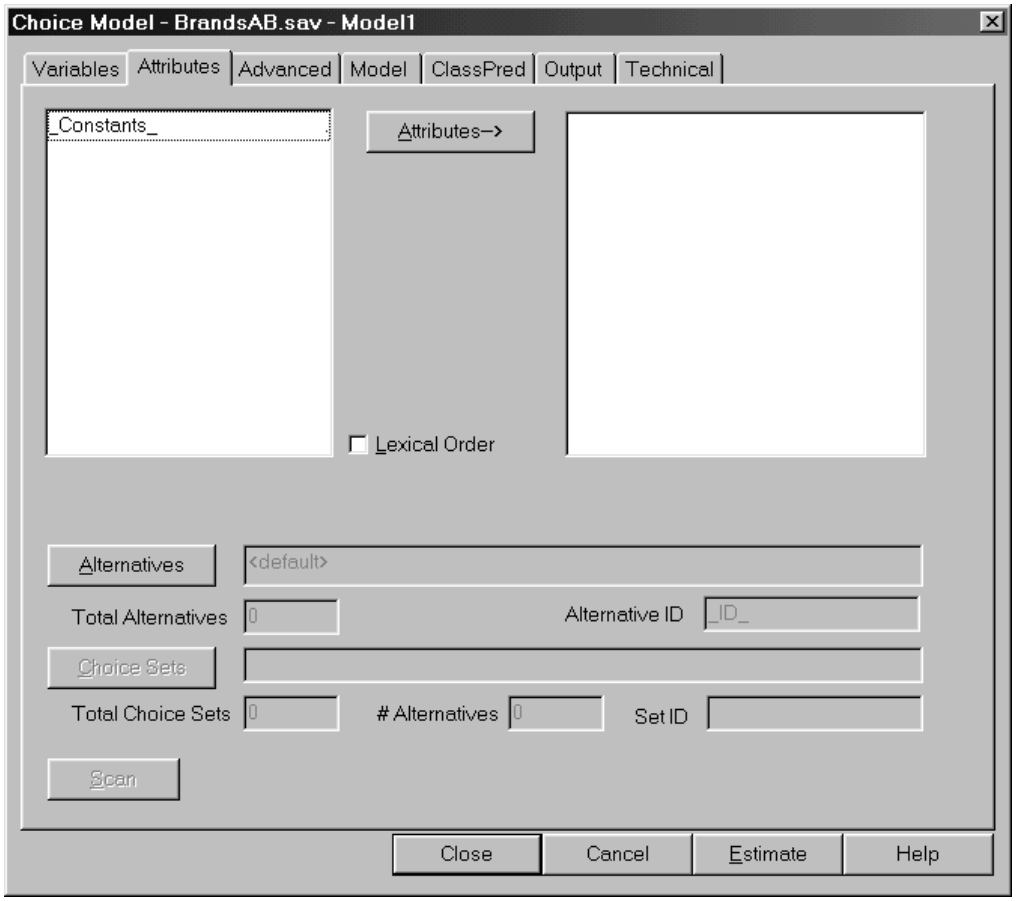



**Figure 6. Attributes Tab**

The alternative-specific constants '_Constants_' appear in the Variable List Box along with any other attributes.

**Alternatives Button** – click this to open an alternatives file (see Tutorial #1 for an example). The specified Alternative ID variable name is displayed in the **Alternative ID** box.

**Choice Sets box.** This box becomes activated after an Alternatives file has been opened. The specified Set ID variable name is displayed in the **Alternative ID** box.

**Attributes box** – To estimate an effect for an attribute, select it from the List box and move it to the Attributes box. Right click on any attribute in the Attributes box to set the scale type (Nominal or Numeric). Note: the '_Constants_' may not be set to Numeric.

**Scan** Upon clicking Scan, the Alternatives and Sets file are scanned and values for the following are displayed:

Total Alternatives
Choice Sets
Total Choice Sets
#Alternatives

Prior to estimating the model you may also wish to pre-specify certain restrictions that you wish to impose on the model parameters. Open the Model Tab to apply restrictions. You may also alter the technical settings (in the Technical Tab).

## Advanced Tab (Advanced Module only)

The Advanced Tab is divided into 3 sections according to the labels 'Survey', 'Continuous Factors', and 'Multilevel Model'.

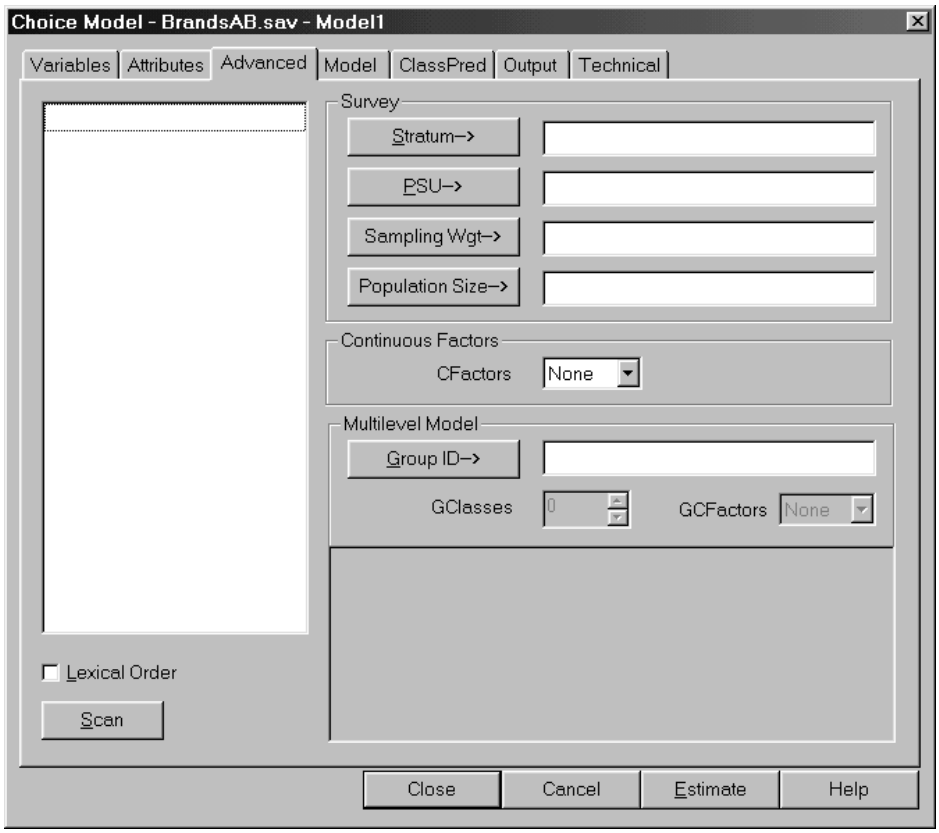



**Figure 7. Advanced Tab**

The variables displayed in the variable list (left-most box of the Advanced Tab) are those that have not been specified previously for use previously as an attribute, dependent variable, predictor, covariate, known class indicator, case ID, case weight, or replication weight. These variables are eligible for use with any of these 3 advanced options.

**Survey**

This advanced option can be used to specify information on the sampling design that was used to obtain your data. The program computes the design effect, as well as reports sampling design corrected standard errors and Wald statistics. Four aspects of the sampling design can be taken into account: stratification (Stratum), clustering (PSU), weighting (Sampling Wgt), and finite population size (Population Size).

For more details, see section 8 of the Technical Guide.

**Stratum**

The Stratum variable specifies the stratum to which a case belongs. When no Stratum variable is specified, it is assumed that all cases belong to the same stratum; that is, that there is only one stratum.

**PSU**

The PSU(Primary Sampling Unit) variable is used for two-stage cluster samples. It specifies the (sampling) cluster to which a case belongs. PSUs are assumed to be nested within strata. When no PSU variable is specified, it is assumed that each case forms a separate PSU.

**Sampling Wgt**

The Sampling Wgt variable contains a sampling weight.

**Rescale (default) vs. No Rescale**. Upon selecting a variable to be used as a Sampling Wgt, the symbol <R> appears in the Sampling Wgt box to the right of the variable name to indicate that the weights will be rescaled. Rescaling of the original weights are accomplished by multiplying them by a constant such that the sum of the weights equals the sample size.

- Right click on the variable name and select 'No Rescale' from the popup menu to maintain the weights without rescaling. Upon selection of 'No Rescale', the <R> symbol is removed.

**Active (default) vs. Inactive**. By default the sampling weights are used in the estimation to compute pseudo maximum likelihood estimates as indicated in Section 8.1 of the Technical Guide. If the sampling weight were instead specified as a Case Weight in the Variables Tab, the resulting parameter estimates would be the same as when the Active option is used here but the standard errors are not correct.

The inactive option for sampling wgt employs an alternative 2-step estimation algorithm developed by Vermunt and Magidson (2001). .

- Right click on the variable name and select 'Inactive' from the popup menu to select this option. Upon selection of 'Inactive', the <I> symbol appears in the Sampling Wgt box to the right of the variable name.

If the sampling weight variable were instead not used at all in the estimation (not specified as either a Case Weight nor a Sampling Wgt), the parameter estimates obtained would be the same as when the Inactive option is used here, but the sizes of the latent classes would be biased. The advantage of this method over the Active option is that the unweighted estimates may be more stable. See Section 8.2 of the Technical Guide for further information about these options.

**Population Size**

The Population Size variable can be used to specify either the size of the population (# of PSUs in the Stratum concerned) or the population fraction. The variable is assumed to be a population fraction when it is smaller or equal to 1. This option can be use for finite population corrections.

## Continuous Factors

This advanced option can be used to include up to 3 continuous latent variables (CFactors) in a model, yielding standard random-coefficients discrete-choice model and LC discrete-choice models with random coefficients.
CFactors are assumed to affect the alternative-specific constants, as well as the attribute and predictor effects, which implies that the corresponding parameters are random coefficients whose values vary across cases. On the Model tab, one can exclude some of the CFactor terms, and when combined with latent classes, one can use the standard between class parameter constraints.

By default, the CFactors box is set to 'None'. To include CFactors in a model, click to open the drop down menu and select the number of CFactors to include in the model (1, 2, or 3). When 1 or more CFactors are included in the model, they appear on the Model Tab for further model specification.

By default, CFactors use 10 nodes to approximate normally distributed variables. To improve precision of the estimates, the number of nodes may be increased to a value as high as 50 (or reduced as low as 2). This change is made in the Continuous Factors section of the Technical Tab.

Warning: Inclusion of CFactors in a model may substantially increase the amount of time required to estimate the model. For example, inclusion of 2 CFactors, results in 10x10 = 100 nodes used to

approximate the bivariate normal distribution for these CFactors. Increasing the number of nodes to 50 results in 50x50 = 2500 nodes, which will substantially increase the amount of estimation time. For further details, see Section 6 of the Technical Guide.

When used with a 1-class model, the result is not a LC model but a standard random-coefficient conditional logit model . For further details regarding the various kinds of applications with CFactors, see Section 6 of the Technical Guide.

## Multilevel Model

This advanced option is used to specify a multilevel extension to an LC Choice Model which allows for explanation of the heterogeneity not only at the case level, but also at the group level.

Heterogeneity at the group level is explained by the inclusion of group-level classes (GClasses) and/or group-level CFactors (GCFactors) in a model.

**Group ID**

The Group ID variable indicates to which higher-level unit or group each case belongs. Upon selecting a variable as the Group ID, the Group Specification Box in the lower-right portion of the Advanced Tab is activated.

**GClasses**

This option assumes that groups belong to one of a set of latent classes of groups, the number of which is specified with GClasses (Group-level Classes). This yields the nonparametric variant of the multilevel LC model. By default, the GClass box is set to 1. To use this option, specify 2 or more GClasses

Click the up arrow in the drop-down box to increase the number of GClasses to 2 or more (up to 100). The GCFactors then appear in the Group Specification Box below.

**GCFactors**

This option assumes that groups differ with respect to their scores on one or more group-level continuous factors (GCFactors) or group-level random effects. This yields the parametric variant of the multilevel LC model. Click on the drop-down box to select the number of GCFactors. The GCFactors then appear in the Group Specification Box below.

GClasses and GCFactors may both be specified to combine the parametric and nonparametric approaches.

GClasses and GCFactors may
affect the intercept and the covariate effects in the model for the Classes
affect the alternative-specific constants and the attribute and the predictor effects in the model for the dependent variable (see Model Tab )

GClasses may themselves be affected by Group-level covariates (GCovariates).

When CGClasses or CGFactors are included in the model, they appear on the Model Tab for further model specification as described earlier; that is, to indicate whether alternative-specific constants, attribute effects and predictor effects vary (randomly) across groups. To include GClass and/or GCFactor effects in the model for the Classes, use the Group Specification Box.

**Group specification box**

The Group specification box at the lower-right of the Advanced Tab contains a column for GClasses and additional columns for each GCFactor specified. Click in the check boxes to allow estimation of desired parameters.

When GClasses and/or GCFactors are included, it is assumed that these affect the intercept in the Model for the Classes. This yields the standard multilevel latent choice model in which class sizes are assumed to differ across groups by using a (parametric or nonparametric) random-intercept model for the latent classes.

GClasses and GCFactors may also be allowed to affect the covariate effects in the Model for the Classes. This is accomplished by checking the corresponding terms on the Advanced Tab.

GClasses play a role similar to the one of the Classes in the discrete-choice model: effects can be GClass independent (check off) or GClass dependent (check on). GCFactors play a role similar to CFactors in a random-effects choice model: effects can be assumed to be fixed (check off) or random (check on).

See Section 7 of the Technical Guide for details and application types of multilevel models.

## Model Tab

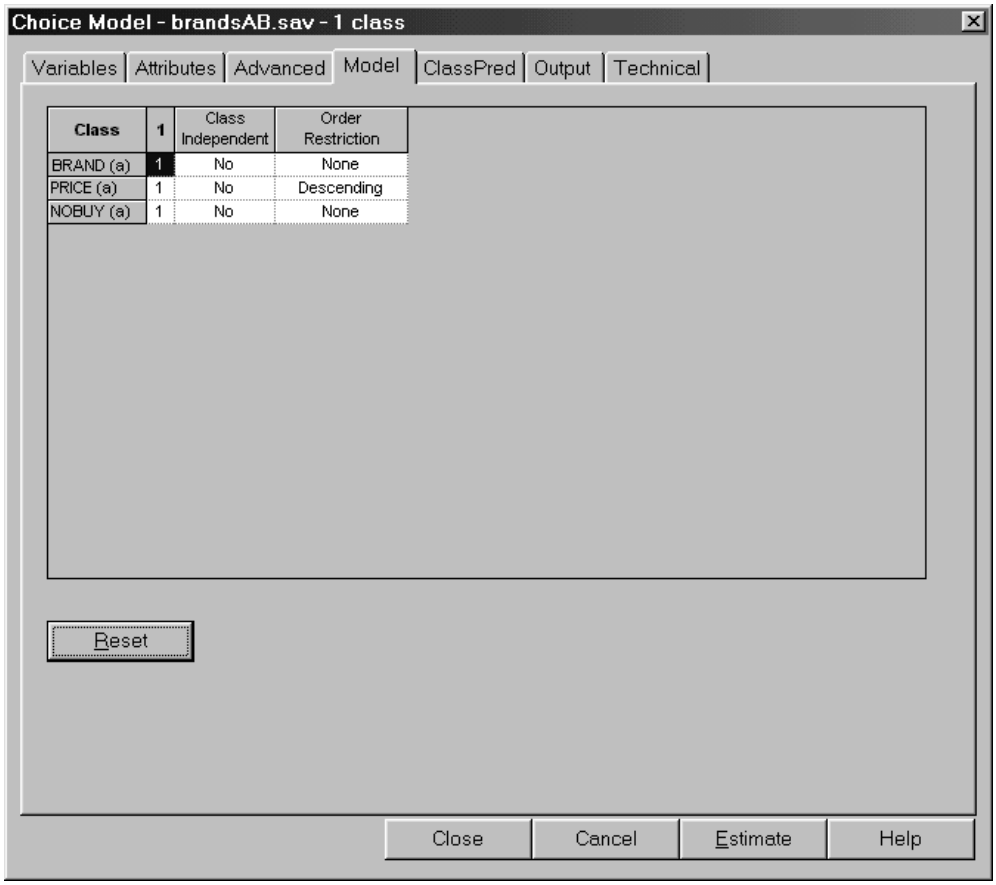
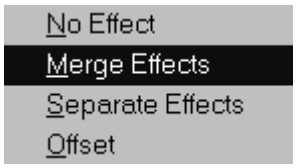



**Figure 8. Model Tab**

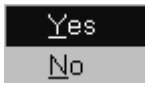
Use this tab to specify different kinds of restrictions These restrictions are available for alternative-specific constants, attribute effects, and predictor effects. These terms can be set equal to zero for certain classes (*No Effect*), equated across two or more classes (*Merge Effects*), and equated across all classes (*Class*

*Independent*). The effect of an attribute or a predictor can be restricted to be ordered in each class (*Ascending or Descending*).

These constraints are activated by means of a pop-up menu that appear following a right click on the desired cells in the grid. For example, a right click on one or more selected cells in the first set of columns corresponding to the classes brings up the menu:




Right clicking on one or more cells under the column marked 'Class Independence' brings up the following menu:




To impose the restriction of *class independence*, select one or more cells containing the 'No' label, right click and select 'Yes' from the pop-up menu. The 'No' changes to 'Yes' in the selected cells, and the indices in the selected rows all change to '1' to indicate that the effects for classes 2, 3, etc. are all restricted to be equal to the corresponding effects for class 1. Alternatively, the restriction of *class independence* can be imposed as follows: 1) select all the cells containing the indices for a chosen row, 2) right click to bring up the pop-up menu, and 3) select 'Merge Effect'. To undo these restrictions, reselect the cells, right click and select 'Separate Effect'.

For rating models, the class independence restriction has the additional option "No-Simple" for the _Contants_. This yields a more parsimonious model for describing the manner in which constants differ across classes.
Right clicking on one or more cells under the column marked 'Order' brings up a menu that allows the imposition of an order restriction (Ascending, Descending or None)

Click the **Reset** button to reset the restrictions to the original default settings of no restrictions.

By default, no restrictions are imposed (i.e., Separate Effects are estimated for each parameter). For further details see section 2.11 in the Technical Guide.

## ClassPred Tab

The **ClassPred Tab** contains various output-to-file options associated with prediction and classification, and also contains a **Known Class indicator** option which allows more control over the segment definitions by pre-assigning selected cases (not) to be in a particular class or classes.

### Known Class option: Restricting Cases Known (Not) to Belong to a Certain Class or Classes

With this option one can specify that one or more specific cases can belong to a certain class or certain classes only. To use this feature, select a variable from the list box in the ClassPred Tab to be used as the Known Class Indicator and click Known Class. The variable moves to the Known Class Indicator Box and the Assignment Table becomes active. For each category of the Known Class indicator you then specify to

which classes the cases with that category code may belong *(or not belong)* using the Assignment Table. For example, Figure 9 illustrates a 4-Cluster model (4 columns) where the variable 'classind' is used as the Known Class Indicator. Cases for which 'classind=1' are allowed to be in cluster 1 only; those for which 'classind=2' are allowed to be in cluster 2 only; all other cases ('classind=3') may be assigned to any of the 4 clusters.

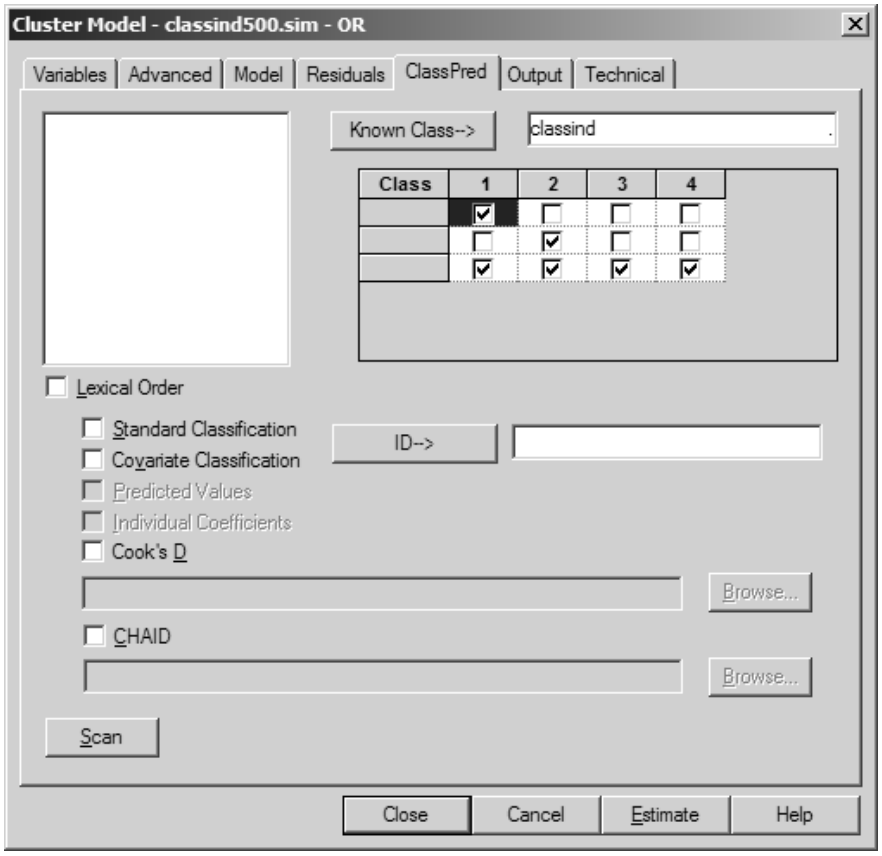



**Figure 9. ClassPred Tab for Cluster Model with a Known Class Indicator**

This option is useful if you have a priori class membership information for some cases (pre-assigned or pre-classified cases) or if membership to certain classes is very implausible for some combinations of observed scores.

## Known Class - Class Indicator

In applications where a subset of the cases are known with certainty not to belong to a particular class, or particular classes, you can take advantage of this information to restrict their posterior membership probability to 0 for one or more classes and hence classify these cases into one of the remaining class(es) with a total probability = 1. This feature allows more control over the segment definitions to ensure that the resulting classes are most meaningful. Common applications include:

1) using new data to refine old segmentation models while maintaining the segment classifications of the original sample
2) archetypal analysis – define class membership a priori based on extreme choice response patterns that reflect theoretical "archetypes"

3) partial classification -- high cost (or other factors) may preclude all but a small sample of cases from being classified with certainty. These cases can be assigned to their respective classes with 100% certainty, and the remaining would be classified by the LC model in the usual way
4) post-hoc refinement of class assignment where modal assignment for certain cases is judged to be implausible based on the desired interpretation of the classes.

In addition, this option may also be used to specify multiple group models by including the group variable as both a Known Class Indicator and as an active covariate. For further details of this, see section 2.5 of the Latent GOLD Technical Guide.

To select known classes:

- Select one variable from those appearing in Variables List Box (located in the upper left-hand portion of the ClassPred Tab). Variables appearing here are those that have not been previously selected.

- Click Known Class to move that variable to the Known Class Box and the class assignment window beneath the Known Class Box becomes active.

- A separate row appears for each category/code/value taken on by the known class indicator
  A separate column appears for each class.

- Click on the appropriate boxes to select or deselect the possible assignment of the categories to certain classes.

A checkmark off means that the posterior membership probability is restricted to zero for that class for cases in that category of the known class indicator.

By default, the checks are assigned as follows:

For a *K*-class model, a category with a code of *K* on the Known Class Indicator is assigned to only class *K*. Categories coded less than 1, greater than *K* or missing are assigned to all classes (i.e., no restrictions). Missing values are not shown in the table.

Note: For the example in Figure 9 above, all cases are coded either 1, 2 or 3 on the variable 'classind' (i.e., no missing values). Those coded 'classind=1' and 'classind=2' are maintained at their default specifications on the table, while the default specification for cases coded 'classind=3' was changed (from 'class 3 only' to 'any class' -- all 4 class columns checked). This specification would be obtained by default if those coded '3' on the classind variable were instead coded as 'missing'. In this situation, the table would differ from that shown in Figure 9, in that the 3rd row of the table would not appear, since that category would be coded 'missing'.

For further information, see section 2.5 of the Latent GOLD Technical Guide.

## Technical Tab

The default settings for the estimation algorithm are shown below. Changes to one or more of these settings can be made in the Technical Tab before or after estimation. A description of the estimation algorithm and various options are given in section 3 of Part 2 of this manual.

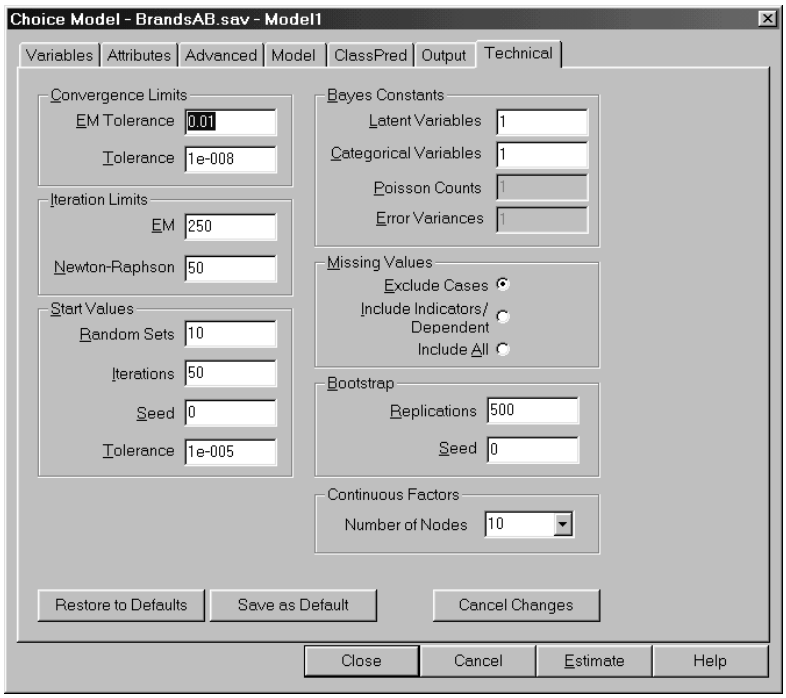



**Figure 10. Technical Tab**

## Convergence Limits

**EM Tolerance**

EM Tolerance is the sum of absolute relative changes of parameter values in a single iteration. It determines when the program switches from EM to Newton-Raphson (if the NR iteration limit has been set to > 0). Increasing the EM Tolerance will switch faster from EM to NR. To change this option, double click the value to highlight it, then type in a new value. You may enter any non-negative real number. The default is 0.01. Values between 0.01 and 0.1 (1% and 10%) are reasonable.

**Tolerance**

Tolerance is the sum of absolute relative changes of parameter values in a single iteration. It determines when the program stops its iteration The default is 1.0x10-8 which specifies a tight convergence criterion. To change this option, double click the value to highlight it, then type in a new value. You may enter any non-negative real number.

## Iteration Limits

**EM Iterations**

Maximum number of EM iterations before switching to Newton-Raphson (if NR iteration is not equal to 0). The default is 250. If the model is estimated using EM only (if you set NR iterations = 0) and it does not converge after 250 iterations, this value should be increased. You also may want to increase this value. To change this option, double click the value to highlight it, then type in a new value. You may enter any non-negative integer.

**Newton-Raphson**

Maximum number of NR iterations. The default is 50. If the model does not converge after 50 iterations, this value should be increased. To change this option, double click the value to highlight it, then type in a new value. You may enter any non-negative integer. A value of 0 is entered to direct Latent GOLD to use only EM, which may produce faster convergence in models with many.

## Start Values

To reduce the likelihood of obtaining a local solution, the following options can be used to either increasing the number of start sets, the number of iterations per set, or both.

**Random Sets**

The default is 10 for the number of random sets of starting values to be used to start the iterative estimation algorithm. Decreasing the number of sets of random starting values for the model parameters reduces the likelihood of converging to a local (rather than global) solution. To change this option, double click the value to highlight it, then type in a new value. You may enter any non-negative integer. Using either the value 0 or 1 results in the use of a single set of starting values.

**Iterations**

This option allows specification of the number of iterations to be performed per set of start values. Latent GOLD Choice first performs this number of iterations within each set and subsequently twice this number within the best 10% of the start sets. For some models, many more than 50 iterations per set may need to be performed to avoid local solutions.

**Seed**

The default value of 0 means that the Seed is obtained during estimation using a pseudo random number generator based on clock time. Specifying a non-negative integer different from 0, yields the same result each time.

If the current model setup was obtained by opening an .lgf file associated with a previously estimated model, 1) the Seed will not be 0 but will be the Best Start Seed for that model as specified in the .lgf file, and 2) the Random Sets parameter will be set equal to 0. This procedure assures that the model estimated is exactly the same model obtained when originally estimated, as long as the .lgf file was created using Latent GOLD 4.0 (see Warning below).

To specify a particular numeric seed (such as the Best Start Seed reported in the Model Summary Output for a previously estimated model), double click the value to highlight it, then type in (or copy and paste) a non-negative integer. When using the Best Start Seed, be sure to deactivate the Random Sets option (using Random Sets = 0). For further details see section 3.6 of the Technical Guide.

Warning: Due to improvements in this option in Latent GOLD Choice 4.0, the random seed obtained from earlier versions of Latent GOLD Choice will not necessarily reproduce the original model and has an increased chance of resulting in a local solution. Hence, if you open an .lgf file that was created using an earlier version of Latent GOLD Choice, you should make sure to restore the default value of 0 and increase the value for Random Sets to the default value of 10 or some other desired quantity.

**Tolerance**

Indicates the convergence criterion to be use when running the model of interest with the various start sets. The definition of this tolerance is the same as the one that use used for the EM and Newton-Raphson Iterations.

## Bayes Constants

The Bayes options can be used to eliminate the possibility of obtaining boundary solutions. You may enter any non-negative real value. Separate Bayes constants can be specified for three different situations:

**Latent Variables**

The default is 1. Increase the value to increase the weight allocated to the Dirichlet prior which is used to prevent the occurrence of boundary zeroes in estimating the latent distribution. The number can be interpreted as a total number of added cases that is equally distributed among the classes (and the covariate patterns). To change this option, double click the value to highlight it, then type in a new value.

**Categorical Variables**

The default is 1. Increase the value to increase the weight allocated to the Dirichlet prior which is used in the model for the dependent variable. The number can be interpreted as a total number of added cases to the cells in the models for the dependent to prevent the occurrence of boundary zeroes. These pseudo cases are divided equally across classes and attribute/predictor/covariate patterns, and in accordance with the observed marginal distribution across categories of the dependent variable. To change this option, double click the value to highlight it, then type in a new value.

## Missing Values

The Missing Values option allows for the inclusion of cases containing missing values on covariates and sets containing missing values on predictors, as well as sets containing missing values on the dependent variable. Including cases with missing values on covariates and sets with missing predictors causes the mean to be inputed for the scale type numeric and the effect of the missing value category to be equated to zero for the scale type nominal. Missing values on the dependent variable are handled directly in the likelihood function. In fact, choice sets with a missing value on the dependent variable do not contribute to the log-likelihood function

Note that missing values on attributes will never cause deletion of choice sets from the analysis.

**Exclude cases**

Selection of this option excludes all replications having missing values on the dependent variable or any of the predictors and all cases having missing values on any of the active covariates.

**Include indicators/dependent**

Selection of this option excludes replications having missing values on any of the predictors and cases having missing values on any of the active covariates.

**Include all**

Selection of this option includes all cases and replications in the analysis regardless of the presence of missing values. Cases or replications with missing values on the dependent variable are included in the analysis and handled directly in the likelihood function. Missing values on Predictors or active covariates are imputed using Latent GOLD's imputation procedure

Inclusion in a model of covariates designated as inactive has no effect on which cases are excluded. Therefore, these missing values options have no effect with respect to the presence or absence of missing values on covariates specified to be inactive

## Bootstrap Options (Bootstrap $L^2$, Bootstrap -2LL Diff)

The Technical Tab contains options for specifying the number of Replications and a Seed for both the Bootstrap $L^2$ and the conditional bootstrap (Bootstrap -2LL Diff) procedures. Either of these bootstrap procedures can be requested from the Model Menu for an estimated model as described in Step 10.

**Replications**

The default for the number of replication samples is 500. In most applications this number is large enough. The program also reports the Monte Carlo standard error of the p-value. By increasing this number, a more precise estimate of the p-value is obtained since the Monte Carlo error is reduced. With large models, to speed up the estimation you may consider reducing the number of replications.

**Seed**

Seed can be used to specify the seed that is used to generate the replication data sets (the default value 0 means random seed) for either the Bootstrap of $L^2$ or the Conditional Bootstrap ('Bootstrap –2LL Diff') procedures. Because of Monte Carlo simulation error, the bootstrap procedure yields a slightly different p-value each time that it is repeated, along with an estimate of the standard error. Specifying a particular seed guarantees the same result each time. By specifying the seed to be equal to the bootstrap seed reported in the Model Summary Output, one can replicate a previous run. In most bootstrap applications one will only use the Replications option.

If the Save Definition option in the File Menu is used to save a .lgf definition file for a model resulting from the Bootstrap, the Bootstrap Seed is saved. For the Conditional Bootstrap, only the Bootstrap Seed associated with the source model is utilized.

## Advanced. Continuous Factors

**Number of Nodes**

If 1 or more (group-level) continuous factors are specified in the Advanced Tab, this option determines the number of nodes used to approximate their normal distribution. By default, 10 nodes are used. Decreasing this number (minimum is 2 nodes) will speed up estimation time but reduce precision of the multivariate normality of the CFactors/GCFactors. For further details see subsections 6.1 and 7.1 of the Technical Guide.

## Default Options

Click **Save as Default** to save the current technical settings as the new default values.

Click **Restore to Defaults** to restore the technical options to their last default values.

Click **Cancel Changes** to cancel any changes that have been made to the Technical options and not saved.

## Step 2: Specify Output Options

Once you have completed your model setup, prior to estimating your model you may wish to specify the output options.

### Output Tab

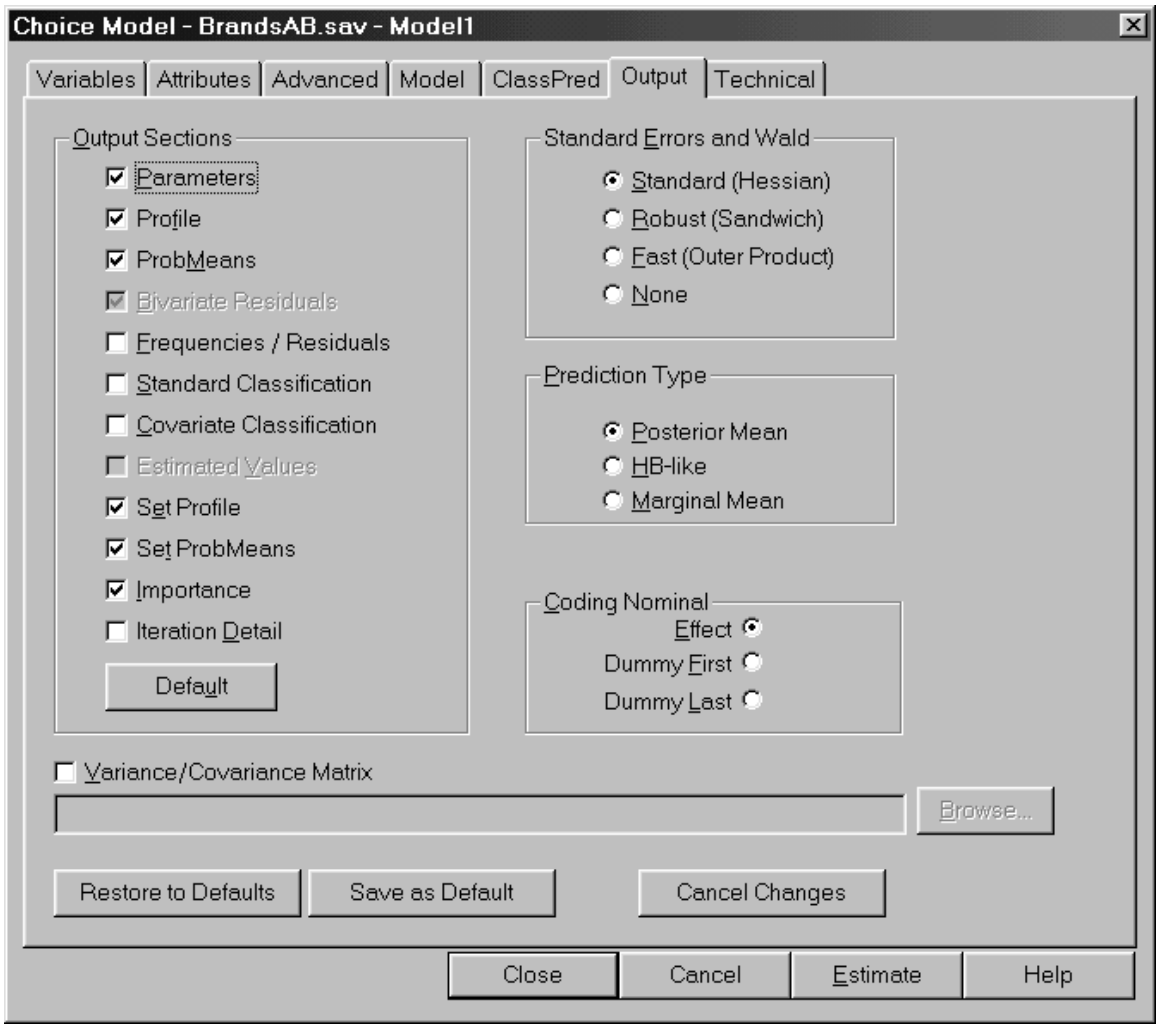



**Figure 11. Output Tab**

A checkmark indicates that the associated Output listings are produced. For details of these output files, see Viewing Output. By default the following are produced (checkmark equals on):

**Parameters**. Shows/hides Parameters in Output Window

**Profile** Shows/hides Profile in Output Window

**ProbMeans** Shows/hides ProbMeans in Output Window

They may be de-selected by clicking the check-box (Output check equals off) in which case this type of output not appear will

The remaining output listings can be obtained by clicking the check-box (Output check equals on).

**Bivariate Residuals**. Produces a Table containing bivariate residuals  The output file will be listed as 'Bivariate Residuals' in the Outline pane.

**Frequencies/Residuals**.  Produces an output file containing observed and estimated frequencies, and standardized residuals for each combination of variables.  This output is not available if any variables in an analysis have been specified as Continuous or Count. This output file will be listed as 'Freqs/Residuals' in the Outline pane.

## Classification Output (optional)

**Standard Classification**. Produces an output file listing containing posterior membership probabilities and other information used to classify cases into the appropriate latent class. This output file will be listed as 'Standard Classification' in the Outline pane.
Each row in the Standard Classification output corresponds to a distinct observed data pattern in the data file.
Advanced: For each CFactor and GCFactors, this file also contains the factor means and for GClasses the classification probabilities and the modal assignment.

**Covariate Classification**. Classification is usually performed based on all available information for a case (Standard Classification). However, it is also possible to compute the probability of being in a certain latent class (or a factor mean), given covariate values only. In fact these are model probabilities; that is, P(x|z) (see Section 4.1.4 of the Technical Guide). These probabilities are useful for classifying new cases for which information on the dependent variable or indicators is not available.
Each row in the Covariate Classification output corresponds to a distinct pattern of active covariates that is observed in the data file.

Note: Inactive covariates do not influence the classification probabilities and hence have no affect on this output.

Standard Classification information as well as Covariate Classification can be viewed as Tabular output and/or can also be output to an external file. Selection of these from the Output Tab produces the Tabular output. Selecting Standard Classification and/or Covariate Classification from the 'Output to File' section of the ClassPred Tab produces the external files which contain the classification information as new variables appended to a copy of the input file used for estimation. See below
**Set Profile**
Shows/hides Set Profile in Output Window.

**Set ProbMeans**
Shows/hides Set ProbMeans in Output Window

The Set Profile and Set ProbMeans output sections contain information on the estimated choice probabilities per choice set. For rankings, these are based on the first choice replications only. For choices and ratings, all replications are used. For more information, see Section 4.5 of the Technical Guide

**Importance**
Shows/hides Importance in Output Window. The Importance output reports the maximum effect for each of the attributes, including the constants, as well as re-scaled maximum effects that add up to one within latent classes. For more information, see Section 4.3 of the Technical Guide

**Iteration Detail** Shows/hides Iteration Detail in Output Window. If this output is not selected, it still will appear if any problems are encountered during model estimation.

## Standard Errors and Wald

Choose one of four options:

The first three options specify different types of information matrices to be used in the computation of standard errors and Wald statistics. The fourth option suppresses such computations

**Standard (Hessian)**

The Standard method makes use of the second-order derivatives of the log-likelihood function called the Hessian matrix. This is the default option.

**Robust (Sandwich)**

The Robust method "sandwiches" the inverse of the outer-product matrix by the Hessian matrix. Standard errors and Wald statistics obtained by the Robust method are less affected by distributional assumptions about the indicators and the dependent variable.

**Fast (Outer Product)**

The Fast method approximates the information matrix using the outer-product of the first-order derivatives of the log-likelihood function. The Fast method may be used in models in which the other two methods are computationally intensive. In such cases, one can also suppress the computation of standard errors and Wald statistics.

**None**

This option suppresses the computation of standard errors and Wald statistics (option None). This option may be useful when estimating models containing an extremely large number of parameters, in which case computation of the second-order derivatives (used in Newton-Raphson, standard error computations and Wald statistics) may take a lot of time. By setting the Newton-Raphson Iteration Limit to 0 and setting Standard Errors and Wald to "None", the estimation process for such large models is much quicker.

## Prediction Type

Latent GOLD Choice also reports Prediction Statistics. Prediction statistics indicate how well the observed choices, rankings, or ratings are predicted by the specified model. For rankings, the prediction statistics are based on first choices only. For choice and rating variables, all replications are used for obtaining the prediction measures. It is also possible to write predicted values to a file. Predicted values can be computed in three different ways.

**Posterior Mean**

Posterior Mean predicted values are defined as weighed averages of the class-specific predicted values using an individual's posterior membership probabilities as weights.

**HB-like**

As in Hierarchical Bayes, the HB-like predicted values are based on the Individual Coefficients, which are weighted averages of the class-specific regression coefficients with the posterior membership probabilities as weights.

**Marginal Mean**

Marginal Mean uses the prior membership probabilities are weights, which means that the observed values on the dependent variable are not used to generate the predictions.

Posterior Mean and HB-like prediction yield similar results. These methods give a good indication of within-sample prediction performance.
Marginal Mean prediction yields much lower R-sq values, but gives a better indication of out-of-sample prediction performance

For more information on Prediction Statistics, see Section 4.1.5 of the Technical Guide.

## Coding Nominal

**Effect (default)** By default, the Parameter Output contains effect coding for nominal variables. As far as the dependent variable is concerned the coding affects the alternative-specific constants and when specifying choice and rank models, it also affects the predictor effects. In addition, the coding affects the effects of nominal attributes and nominal predictors, and in the model for the classes, it affects the parameters for nominal covariates and for the classes. Use this option to change to dummy coding.

**Dummy Last**. Selection of this option causes dummy coding to be used with the last category serving as the reference category.

**Dummy First**. Selection of this option causes dummy coding to be used with the first category serving as the reference category.

## Variance/Covariance Matrix

When the input data file is either an ASCII text file or an SPSS .sav file, this option outputs the variance-covariance matrix of the parameter estimates to an external file.

**Output Filename**. Upon selection of this option, a default filename appears in the box directly below the check-box. Use the browse button to change the filename and/or its save location. The format of the output file will be the same as that of the input file (ASCII or .sav).

The body of the output file contains the variances and covariances of all model parameters. Each row in this output file corresponds to a parameter. The first variable (column) on this file is a string variable 'Location' which provides a unique name for the parameter such as 'r0001c01'. The 3 right-most columns in the output file are variables called 'se', 'param' and 'Label', serve to define the parameters. For example, for this parameter (row), the string variable called Label might contain a label such as 'purpose : 1 || Class1' which means that this is the parameter estimate for with the 1st category of the attribute PURPOSE, associated with class #1. The variables 'param' and 'se' correspond to the estimate and standard error for this parameter as reported in the Parameters Output.

The remaining variables on the file reproduce the parameter names provided in Location and contain the variance/covariance matrix. For example, the entry in row 1 (Location = 'r0001c01') and column 'r0001c01' is the variance of this parameter estimate. The entry in row 1 (Location = 'r0001c01') and column 'r0001c02' is the covariance associated with parameter estimates 'r0001c01' and 'r0001c02'.

Note: Most users will not need to use this option. These quantities are useful in computing the standard error of a particular function of the parameter estimates. For further details see Sections 3.4 and 4.8 of the Technical Guide.

### Default Options

Click **Default** to restore the Output options to their original program default values.

Click **Save as Default** to save the current output settings as the new default values.

Click **Restore to Defaults** to restore the Output options to their last saved default settings.

Click **Cancel Changes** to cancel any changes that have been made to the Output options and not saved.

## ClassPred Tab

The options on the ClassPred tab can be used to request that certain items should be written to an output files.

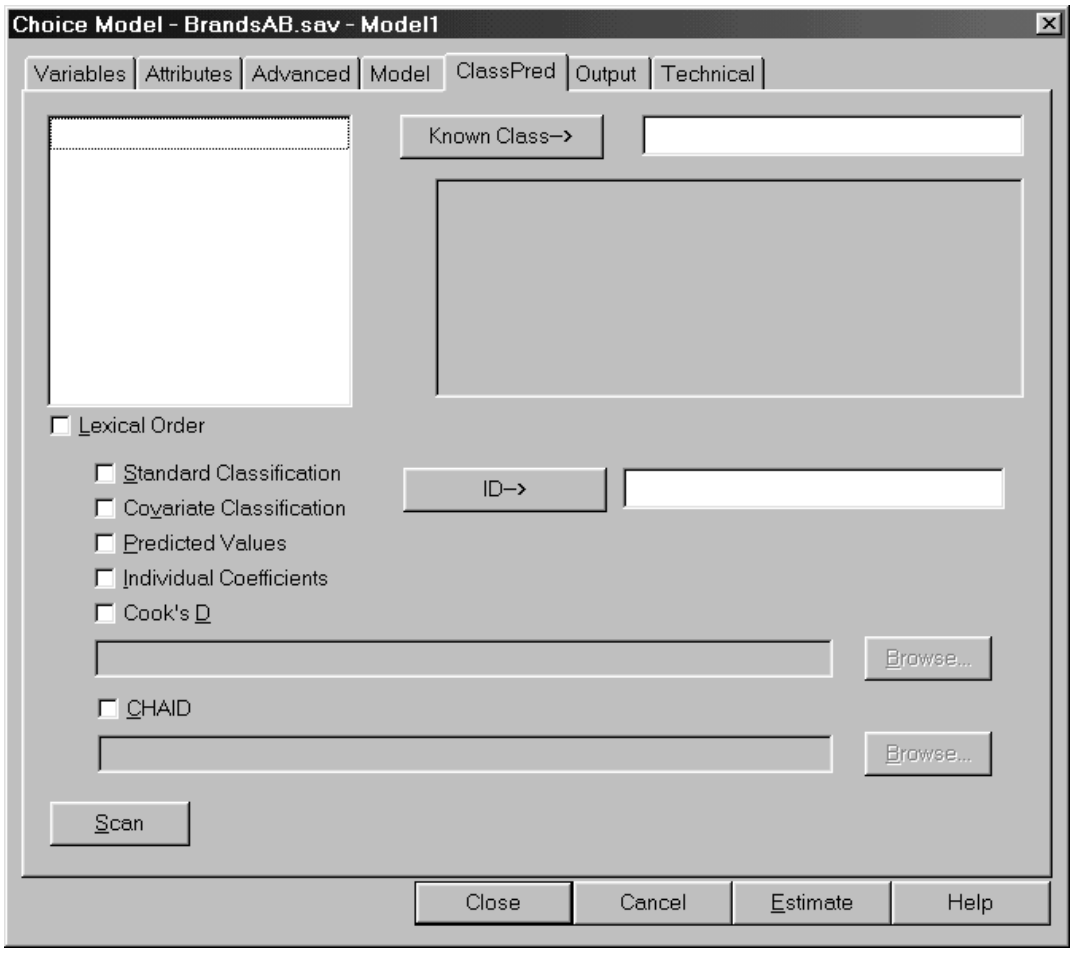



**Figure 12. ClassPred Tab**

### Standard Classification (optional)

When the input data file is either an ASCII text file or an SPSS .sav file, this option produces a new data file containing the standard classification information such as the probability of being in each class together with any covariates and other variables specified in the Variables Tab and the variable included in the ID box of the ClassPred Tab (if any). The format of the output file will be the same as that of the input file (ASCII or .sav). This option is not available when using the range option in the Variables Tab to specify a range of models. In addition to these probabilities Latent GOLD Choice also appends classification variables containing that class into which the respondent should be classified (the one being the highest membership probability). For each case in the analysis file, the variables on the new data file consist of the model variables, the posterior class membership probabilities, and the modal class classification (i.e., the number of the class having the highest posterior probability). As an option, an ID variable can also be appended to the new file. For an example using this option, see Latent GOLD 4.0 Tutorial #3.

Output Filename. A default filename will appear in this box. Use the browse button to change the filename and/or its save location.

**ID Variable**. A single additional variable may be selected for inclusion (typically an ID variable or other key variable which provides a unique identification of each case on the file) to allow additional variables on the original data file that were not included in the analysis to be merged onto this file.

Note: The new file is created after the model has been estimated. After selecting this option, click Estimate to estimate the model and create the new file.

Warning! This setting is not preserved across models – it must be selected explicitly for each model estimated.

**Advanced**: For each CFactor and GCFactor, the corresponding factor means are output; for GClasses, the classification probabilities and the modal assignment are output.

The order of the variables in the output file (and labels for a .sav formatted file) are as follows:

- the Known Class Indicator (if specified in the ClassPred Tab)
- any covariates included in the model
- other model variables specified in the Variables Tab
- the variable included in the ID box of the ClassPred Tab (optional)

- clu#1 (Class1) – posterior membership probability for class 1
- clu#2 (Class2) – posterior membership probability for class 2
- …… (repeated for classes 3,4, …)
- clu# (Class modal) – the class number for the most likely (modal) class

Advanced:

- cfactor1 (‘CFactor1’)
- cfactor2 (‘CFactor2’)
- cfactor3 (‘CFactor3’)
- gclass1 (‘GClass1’)
- gclass2 (‘GClass2’)
- …
- gcfactor1 (‘GCFactor1’)
- …

## Covariate Classification

Classification based on covariates, as is the case with Standard Classification information can be output to an external file. Selecting Covariate Classification from the 'Output to File' section of the ClassPredTab produces the external files. The external file corresponding to the Covariate Classification information contains the new variables appended to a copy of the input file used for estimation.
Advanced: For multilevel models, this output file also contains the GClass probabilities given group-level covariates.

### Prediction Output to a File

**Predicted Values**

Predicted values for the dependent variable can be output to an external file. The method used to determine the predicted values (pred_dep in output file) depends on the Predicted Values setting on the Output Tab (Posterior Mean, HB-like, or Marginal Mean). The predicted value is either the mode (choice and ranking) or the mean (rating) . Also, complete category-specific probabilities are provided for each choice set.

**Individual Coefficients**

It is also possible to output posterior mean estimates for the Individual Coefficients to an external file These are weighted averages of the class-specific effects, where the posterior membership probabilities of a case serve as weights. In the output file, the coefficients appear in the same order as in the Parameters Output and are labeled as b1, b2, b3, etc. Both the individual estimates (est_b1, est_b2, etc) and the individual standard deviations (std_b1, std_b2, etc) are provided.

For more information, see Section 4.1.5 of the Technical Guide

### Cook's D

The Cook's Distance measure may be output to an external file. This measure is used to identify cases that have a large influence on the parameter estimates. A recommended cut-off point for Cook's distance is four times the number of parameters divided by the number of observations.

### CHAID (Requires a license to SI-CHAID 4.0)

This option creates a CHAID settings file (.chd file) from your model that can be then opened via the SI-CHAID 4.0 program. With this option, a CHAID (CHi-squared Automatic Interaction Detector) analysis may be performed following the estimation of any LC model in Latent GOLD 4.0. By selecting 'CHAID' as one of the output options, a CHAID input file will be constructed upon completion of the model estimation, which can then be used as input to SI-CHAID 4.0. For more information regarding CHAID, see page 2.

For additional information on the output to file options, see Sections 4.8 and 4.9 of the Technical Guide.

## Step 3: Estimate the Model

Upon completing the model setup, click Estimate and the model parameters will be estimated. The status bar displays messages regarding the status of the estimation.

Upon beginning a model estimation, the stop button on the toolbar becomes red (this may take several seconds or longer) which indicates that it is now active. You can stop a model estimation once it has begun to accomplish either 1) canceling the estimation, or 2) pausing the estimation to view preliminary results

and/or make changes to the requested output options or change the iteration or convergence limits prior to resuming estimation.

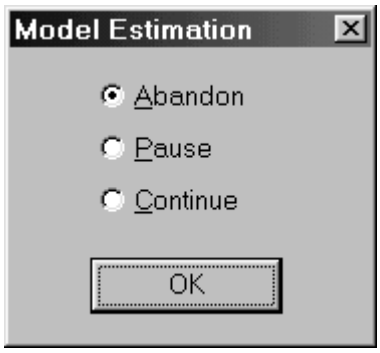
Once the stop button becomes active,

- To stop the estimation procedure select Stop from the Model Menu

or,

- click on the button in the toolbar

and a popup-menu appears:

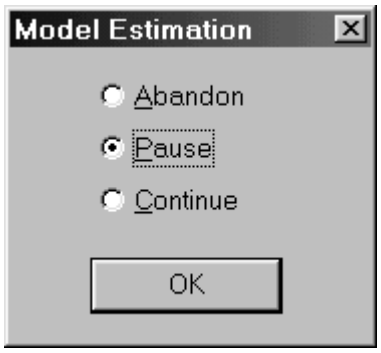



## Abandon/Cancel Estimation of a Model

- Select *Abandon* to cancel the model estimation

This option returns the program to its state prior to beginning the Estimation. Output is produced only for models that completed the estimation process without being terminated.

## Pause Model Estimation




The Pause option allows you to pause a model after the model estimation process has begun but prior to the estimation being completed to review preliminary Model Summary Output as well as any of the following Model Output Sections that were requested in the Output Tab:

Parameters
Profile
ProbMeans
Bivariate Residuals
Iteration Detail

The Pause option is not available during estimation involving a bootstrap procedure.

If you are estimating a range of models, the pause option will pause the estimation process for the model currently being estimated and will cancel any further models. Depending upon how large the model is that is being estimated, it may take anywhere from one second to several minutes (or longer) to generate the preliminary output listings.

The Pause option does *not* cause a preliminary version of output to a data file to be created even if such was requested using the ClassPred Tab. If such has been requested, it will be produced only if the estimation is resumed and allowed to complete. See Resuming Estimation of a Paused Model below.

Prior to resuming model estimation, you may modify the Output options that were requested in the Output Tab or change the Iteration or Convergence Limits in the Technical Tab.

After a model is paused, the model name associated with that model appears in the Outline Pane with the characters 'Paused' appended to it. To change the Output options or the Iteration or Convergence limits, double click the model name to open the analysis dialog box and make the desired changes in the Output and/or Technical Tab. Note that the label 'Resume' replaces 'Estimate' on the Estimate button.

### Continue Model Estimation

If Stop was selected in error, click Continue to continue estimating the model.

### Resuming Estimation of a Paused Model (Model Menu)

After viewing the preliminary output and making changes to the options as described above, to Resume the estimation of a paused model:

- select Resume from the Model Menu
  or,
- Click the Resume button at the bottom of the Analysis Dialog Box associated with the Paused model.

- To open the analysis dialog box for a Paused model double click the name of the paused model. In the Analysis Dialog box for a Paused model, the word 'Resume' replaces the word 'Estimate' on the Estimate button.

or, you may

- Click on the name of the Paused Model and select Resume from the Model Menu.

After constructing the various tables needed to initialize the estimation algorithm, the red Cancel light is illuminated and the estimation algorithm begins. To Cancel the estimation prior to completion, click the Red Cancel button. To estimate a new model, simply double click on the last model name and click Estimate button.

# STEP 4: Viewing Output

## Output Options

Once the model has been estimated, the left-hand column (Outline Pane) will display a number of output options. They are

Parameters
Importance
Profile
ProbMeans
Set Profile
Set ProbMeans

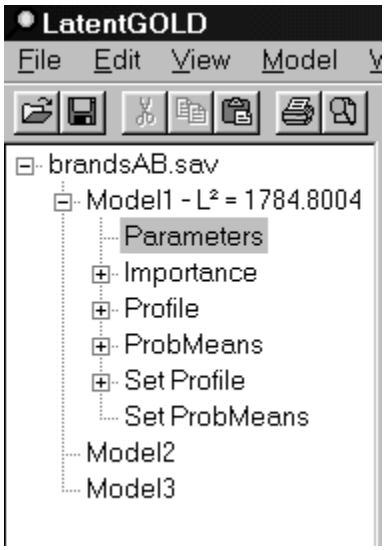



**Figure 13. Output Options**

## Parameters Output

For any estimated model, click Parameters and Latent GOLD displays a table containing parameter estimates and measures of significance for these estimates. For a detailed explanation of these parameters, see Section 4.2 of the Technical Guide.

### Viewing Wald Statistics and Standard Errors

By default, Wald statistics are provided in the output to assess the statistical significance of the set of parameter estimates associated with a given variable (across all classes). Specifically, for each variable, the Wald statistic tests the restriction that each of the parameter estimates in that set equals zero (for variables specified as Nominal, the set includes parameters for each category of the variable). Two Wald statistics (Wald, Wald(=)) are provided in the table when more than 1 class has been estimated. For each set of parameter estimates, the Wald(=) statistic considers the subset associated with each class and tests the restriction that each parameter in that subset equals the corresponding parameter in the subsets associated with each of the other classes. That is, the Wald(=) statistic tests the equality of each set of regression effects across classes.

- To view standard errors (or Z-statistics) associated with parameter estimates, from the menus choose:

  View

  Standard Errors (or Z statistic)

and the column containing the Wald statistic(s) is replaced by the standard errors (Z statistics). Standard errors can be used as a diagnostic tool to assess the significance of a *single* parameter estimate. As a rule of thumb, parameter estimates larger than twice their standard error are significant at the .05 level.

- To replace the standard errors by the Wald statistic(s) again, from the menus choose

  View

  Wald

## Importance

The Importance output reports the maximum effect for each of the attributes, including the constants, as well as re-scaled maximum effects that add up to one within latent classes (relative importances). The latter are plotted in the importance plot (Imp-Plot).

## Profile View

- To view the profile table for a selected model, click Profile in the Outline pane. For Attributes (and the Contants), this output section contains the class-specific part-worths that are transformed by an inverse logit transformation, and thus sum to 1 across attribute levels within classes. For covariates, these are rescaled ProbMeans probabilities. For a detailed interpretation of this output, see Section 4.4 of the Technical Guide.
- Profile Plot
- To view the corresponding Profile Plot, click the '+' icon to expand the Profile output and highlight Prf-Plot.
- Click on any variable symbol in the Profile Plot and the status bar describes it (variable name, class/factor level number, value).
- Click on any class name or symbol in the legend and Latent GOLD highlights all the symbols that refer to that class/factor level.

### To Change Settings for a Profile

To change the settings for a Profile Plot, right click (or select Plot Control from the Model Menu) within the Contents pane when a Profile Plot is displayed to open the Plot Control dialog box. To change the font type/size for a plot, see Main Menu Options.

# Profile Plot Settings

**Legend**. When this option is selected, a legend appears at the bottom of the Profile Plot.
**Classes.** A line will be drawn for each class selected. Those classes with a checkmark in the checkbox are included in the plot.
**Variables.** Select which variables (Contants, Attributes, Covariates) to include in the plot. Those with a checkmark in the checkbox are included in the plot.
**Categories.** Select which category of a variable to include in the plot. The category currently being plotted is listed in the plot beneath the variable name. To change the category that is plotted, highlight the variable name in the Variables box (the category currently being plotted will appear in the Category box), click the drop-down list to the right of the Categories box and select the category you wish to have plotted.
**Groups:** Click Update once you have specified a new number of groups.

## ProbMeans View

- The table contains class membership probabilities which are displayed in the Uni-Plot and Tri-Plot .

To view the Probability/Means table for a selected model, click ProbMeans in the Outline pane. To view a plot click on the expand '+' icon to the left of ProbMeans to list the type of plots produced for a particular model. Highlight a plot type to view it in the Contents pane.

## Uni-Plot

To view the Uni-Plot, click on the expand/contract icon (+/-) to list the ProbMeans plots and highlight Uni-Plot. The larger the distance (range) between points belonging to a particular variable, the stronger the variable is related to the latent variable.

- By default, a separate Uni-Plot is created for each class. Symbols appear in the plots for each value of each attribute and choices corresponding to a selected choice set.
- Click on any variable symbol in the Uni-Plot and the plot label will appear and the status bar will contain a description of the point (variable name and category value).
- Click on any variable name or symbol in the legend and Latent GOLD choice will highlight all the points that refer to that variable.

### To Change Settings for a Uni-Plot

To change the settings for a Uni-Plot, right click (or select Plot Control from the Model Menu) within the Contents pane when a Uni-Plot is displayed to open the Plot Contro l dialog box. To change the font for a plot, see Main Menu Options

## Uni-Plot Settings

**Legend**. When this option is selected, a Legend appears at the bottom of the Uni-Plot.
**Point Labels.** When this option is selected, category labels for each variable are listed on the Uni-Plot next to the variable symbol.
**Classes.** Select which Classes to include in the Uni-Plots. For each class selected (a checkmark in the checkbox) a Uni-Plot will be displayed. By default, all classes are selected.
**Axis Flip.** To flip (reverse) the axis for a Uni-Plot, select the corresponding class/factor name. By default, the class probabilities/factor mean range is from 0 to 1 (increasing). Selecting Axis flip for a class/factor will reverse the axis to range from 1 to 0 (decreasing).
**Variables.** Select which indicators/covariates to include in the Uni-Plots. Selected variables are indicated by a checkmark in the checkbox. By default, the Uni-Plots contain all the indicators/covariates included in the model.
**Set** Can be used to indicate which choice sets should appear in the be plot.
**Groups**. Use the grouping option to reduce the number of categories for a variable, click Update once you have specified a new number of groups for further details on the grouping option).

## Tri-Plot

The probabilities in the body if the ProbMeans output table are plotted to form a Tri-Plot. To view the Tri-Plot, click on the expand/contract icon (+/-) to list the ProbMeans plots and highlight Tri-Plot. (Note: No Tri-Plot is produced for a 1-class model; for a 2-class model, the Tri-Plot reduces to the Uni-Plot.)

- By default, Vertex A (left-most base vertex) is labeled 'Class 1', Vertex B (right-most base vertex) 'Class 2', and the third Vertex (the top point of the triangle) represents the aggregate of all other classes. For a 3-class model, by default, the third vertex will

represent Class 3 and is labeled 'Class 3'. For a 4-or-more class model, the third vertex is labeled 'Others'. For a 2-class model, the class 3 membership probability is 0 and the Tri-Plot reduces to the Uni-Plot.

- The triangle symbol marks the overall probabilities for the 3 classes associated with the vertices. It represents the centroid of the triangle.

- Click on any variable symbol in the Tri-Plot and 1) the status bar will contain a description of the point (variable name and category, class probabilities) 2) the category label will appear next to that point on the plot and 3) lines emanate from that point to each side of the triangle, intersecting the side at the corresponding class probabilities value.

- Click on any variable symbol or name in the legend and all the symbols for that variable will be highlighted and their category labels listed in the Tri-Plot.

### To Change Settings for a Tri-Plot

To change the settings for a Tri-Plot, right click (or select Plot Control from the Model Menu) within the Contents pane when a Tri-Plot is displayed to open the Plot Control dialog box. To change the font for a plot, see Main Menu Options
This is a graphical presentation of the data presented in the ProbMeans view.

## Tri-Plot Settings

**Legend**. When this option is selected, a Legend appears to the right of the Tri-Plot.
**Point Labels**. When this option is selected, category labels for each variable are listed on the Tri-Plot next to the variable symbol.
**Vertexes**. Latent GOLD Choice allows you to select the base vertices in the Tri-Plot. The top vertex corresponds to the aggregate of the remaining classs.

- A vertex. The class currently used as the A vertex is listed in the drop down box. To select a different class, click on the down arrow to the right of the vertex box. A drop list containing all class will appear. Select the class to use as the A vertex.

- B vertex. The class currently used as the B vertex is listed in the drop down box. To select a different class, click on the down arrow to the right of the vertex box. A drop list containing all classs will appear. Select the class to use as the B vertex.

**Variables**. Select which variables to include in the Tri-Plot. Those with a checkmark in the checkbox are included in the plot. By default, the Tri-Plot contains all the indicators/covariates that were input as part of the model.
**Groups**. Click Update once you have specified a new number of groups.

## Set Profile and Set ProbMeans

The Set Profile and Set ProbMeans output sections contain information on the estimated choice probabilities per choice set. For rankings, these are based on the first choice replications only. With choices and ratings, all replications are used.

Set Profile also contains information on the observed choice probabilities as well as residuals per alternative and per set that compare observed with overall estimated choice probabilities.

# Main Menu Options

The Menu Bar in Latent GOLD has 6 general menu options: File, Edit, View, Model, Window, and Help

## File

This file menu can be used to perform the following functions:

**Open**. Opens a data file. Latent GOLD accepts as input data an SPSS system file, an ASCII text (rectangular) file, a special 'array' file format for multi-way tables. In addition, you can Open a previously saved Latent GOLD definition (.lgf) file. Upon opening a data file, the data file name is listed in the Outline Pane (outermost level) and the default file name 'Model1' appears beneath the data file (second level), and may be used to specify and estimate a New Model.

**Close**. Closes the data file highlighted in the Outline Pane.

**Save Results**. Allows you to save your output to either a html or an ASCII (text) file.

**Save Definition**. This saves the analysis settings that have been specified for one or more models on a particular data file.

**Print**. Prints output obtained after any model estimation

**Print Preview.** Preview printed output on screen.

**Print Setup.** Sets various printing options.

At the bottom of the File menu, recently opened data files are listed for easier access.

**Exit**. Exit the program. Prior to exiting, Latent GOLD will prompt you to save your Model definitions and results if you have not already done so.

## Edit

**Copy**. Allows you to copy to the clipboard any output highlighted in the Contents Pane

**Select All**. Selects all output shown in the Contents Pane

**Plot Font**. Allows you to customize the plot font for the output plots in the Content Pane. Upon making a change to the font, this change goes into effect the next time the plot is opened in the Contents Pane.

**Text Style**. Allows you to customize the text style for the output in the Contents Pane. Upon making a change to the text style, this change goes into effect the next time the output listing is opened in the Contents Pane.

**Format**. Allows you to change the format (General, Fixed and Scientific) and number of digits for numeric values displayed in the output.

## View

The options available in the View Menu change depending upon what is highlighted in the Outline Pane. For example, when a model name is highlighted, the options are:

**Toolbar**. Shows the shortcuts Toolbar.

**Status Bar**. Shows the status bar. The status bar displays various information as the model is being estimated.

**ProbChi**. Opens the ProbChi calculator. This calculator can be used to obtain a p-value for a given chi-square (or vice-versa) for a specified number of degrees of freedom, df.

When an interactive table or plot appears in the Contents pane, the View Menu lists the various options for changing the appearance of the associated output.

## Model

The Model Menu options are organized into 3 sections. The first section contains the options for specifying the type of model to be estimated:

**Cluster.** Specifies a Cluster model to be estimated.

**DFactor.** Specifies a DFactor model to be estimated.

**Regression.** Specifies a Regression model to be estimated.

**Choice.** Specifies a Choice model to be estimated

If model names appear in the Outline pane for one or more models that have already been estimated:

If you click on a model name associated with a previously estimated model, the Model Menu contains a checkmark next to the type of model estimated.
If you click on the name for a *new* model (one that has not yet been estimated), there will be a checkmark next to the last type of model estimated.

Model options appearing in the second section of the Model Menu are

**Estimate.** Estimate the model. Select this option once your new model is fully specified

**Estimate All.** This option may be used when the Outline Pane contains names for 1 or more models associated with a data file that have not yet been estimated. Upon selecting the data file name, the Estimate All menu entry becomes active in the Model Menu. Selection of this option causes all of the associated models that have not yet been estimated to be estimated sequentially beginning with the first such model.

Note: Multiple model names associated with models that have not yet been estimated can occur for a data file only if a previously saved definition (.lgf) file containing the setup for 2 or more models is opened. For details on saving .lgf files, see SAVE DEFINITION in the FILE menu options

**Bootstrap $L^2$ and Bootstrap -2LL Diff.** The Bootstrap option can be used to estimate the p-value for certain estimated models.

**Stop.** The Stop command may be used to pause the estimation prior to completion, or to abandon the estimation completely

**Resume**. If a model is paused (default names for paused models have the characters 'Paused' appended to the original model name -- e.g., 'Model4Paused'), the Resume command may be used to continue the estimation process.

**Delete.** This is used to delete the model name (and any associated output files) from the Outline pane.

## Window

**Split** - Allows you to customize the window split between the Outline and Content Panes

## Help

**Contents**. Lists all the Help topics available

**Help.** Displays context-sensitive help.
**Item Help.** Creates a help cursor that you can point to get help on any particular item in the program

**Register.** Displays your registration code.

**About Latent GOLD**. Provides general information about the program.

Many of the tasks you will want to perform with Latent GOLD Choice utilize menu selections. Shortcuts for menu items are listed to the right of the item. For example, the shortcut for File, Open is Ctrl+O (on your keyboard, hold down the Ctrl key and then press the 'O' key).

In addition, a right click in the Contents Pane frequently causes a control panel or the appropriate menu options to appear. For example, a right click in a graphical display (such as the tri-plot) causes the (Tri-plot) Control Panel to appear which can be used to modify the appearance of the Plot in an interactive manner. A right click in the Parameters Output retrieves a pop-up menu containing the Options from the View Menu which allow you to change the appearance of the output in various ways such as adding a column for standard errors.

# Tutorial #1: Using Latent GOLD choice to Estimate Discrete Choice Models

In this tutorial, we analyze data from a simple choice-based conjoint (CBC) experiment designed to estimate market shares (choice shares) for shoes. In this tutorial you will:

- Set up an analysis
- Estimate choice models that specify different numbers of classes (segments)
- Explore which of these models provides the best fit to the data
- Utilize restrictions to refine the best fitting model
- Interpret results using our 'final' model
- Save results

In tutorial 2, the final model will be used to

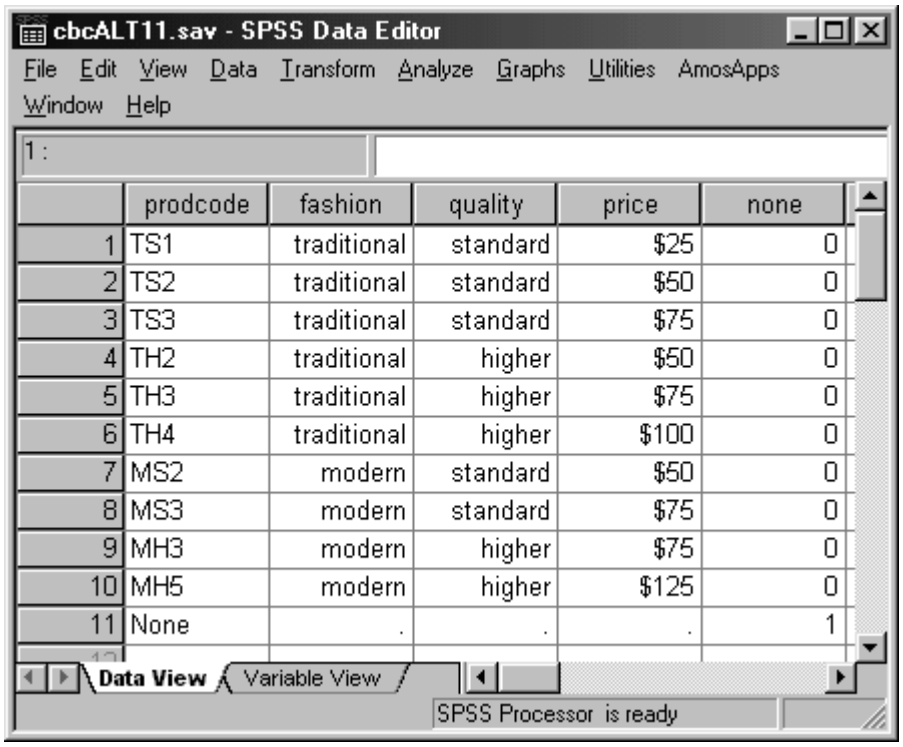
- Predict future choices
- Simulate choices among additional products of interest

## The Data

Latent GOLD Choice accepts data from an optional 1-file or its default 3-file structure from an SPSS .sav file, a Sawtooth .cho file, or ASCII rectangular file format. The current sample data utilizes 3 SPSS .sav files. (An additional 80 file formats are available using the DBMS/Copy add-on).

The 10 pairs of shoes included in this choice experiment differ on 3 attributes: Fashion (0=Traditional, 1= Modern), Quality (0=Standard, 1=Higher), and Price (5 equidistant levels coded from 1 to 5). Each of 8 choice sets offer 3 of these 10 possible alternative products to 400 individuals. The choice task posed to respondents is to assume that each set represents the actual options available for purchase and to select one of these alternatives from each set, with the response "none of the above" allowed as a fourth choice option.

The 11 alternatives (10 different products plus a 'None' option) are defined in terms of the 3 attributes plus the dummy variable NONE in the Alternatives File cbcALT11.sav:

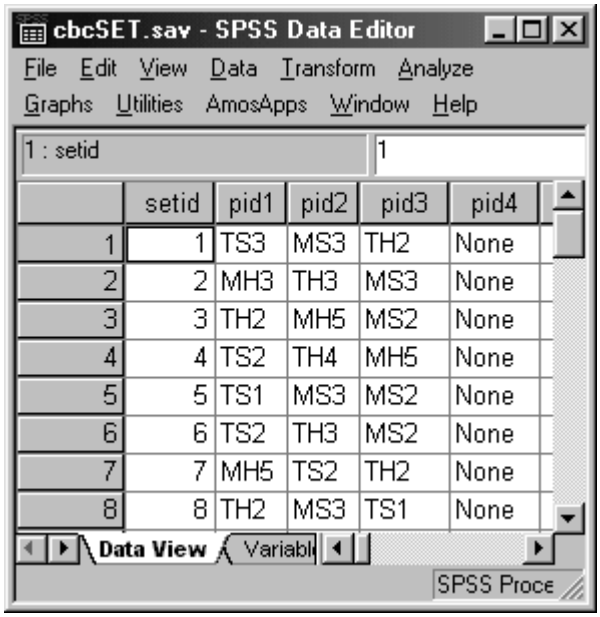



| | prodcode | fashion | quality | price | none |
|---|---|---|---|---|---|
| 1 | TS1 | traditional | standard | $25 | 0 |
| 2 | TS2 | traditional | standard | $50 | 0 |
| 3 | TS3 | traditional | standard | $75 | 0 |
| 4 | TH2 | traditional | higher | $50 | 0 |
| 5 | TH3 | traditional | higher | $75 | 0 |
| 6 | TH4 | traditional | higher | $100 | 0 |
| 7 | MS2 | modern | standard | $50 | 0 |
| 8 | MS3 | modern | standard | $75 | 0 |
| 9 | MH3 | modern | higher | $75 | 0 |
| 10 | MH5 | modern | higher | $125 | 0 |
| 11 | None | . | . | . | 1 |

**Figure 14. Alternatives File 'cbcALT11.sav' defining each of the 11 Alternatives**

The specific 4-alternatives that constitute the 8 choice tasks are defined in the Sets File cbcSET.sav. For example, task 1 involves the choice between shoes TS3, MS3, TH2 or None of these.

**Figure 15. Sets File 'cbcSET.sav' defining each of the 8 choice tasks**




| | setid | pid1 | pid2 | pid3 | pid4 |
|---|---|---|---|---|---|
| 1 | 1 | TS3 | MS3 | TH2 | None |
| 2 | 2 | MH3 | TH3 | MS3 | None |
| 3 | 3 | TH2 | MH5 | MS2 | None |
| 4 | 4 | TS2 | TH4 | MH5 | None |
| 5 | 5 | TS1 | MS3 | MS2 | None |
| 6 | 6 | TS2 | TH3 | MS2 | None |
| 7 | 7 | MH5 | TS2 | TH2 | None |
| 8 | 8 | TH2 | MS3 | TS1 | None |

The universe is assumed to consist of 3 segments with different utility preferences. The first segment constitutes 50% of the Universe, the second and third each representing 25%.

| Segment: | 1 | 2 | 3 |
|---|---|---|---|
| Segment Size | 0.50 | 0.25 | 0.25 |

Part-worth Utility Parameters

| Segment: | 1 | 2 | 3 |
|---|---|---|---|
| Modern fashion | 3 | 0 | 1 |
| Higher quality | 0 | 3 | 1 |
| PRICE | -0.5 | -0.5 | -0.5 |
| NONE | 1 | 0 | -0.5 |

These utilities imply that all 3 segments are equally price sensitive. In addition, segment 1 (50% of the universe) prefers the more Modern style shoes and is indifferent regarding Higher quality, segment 2 (25%) prefers the Higher quality and is indifferent with regards to fashion. Segment 3 (25%) is influenced by both Modern fashion and Higher quality but to a lessor extent than the other segments. Overall, segment 1 is most likely and segment 3 least likely to choose None.

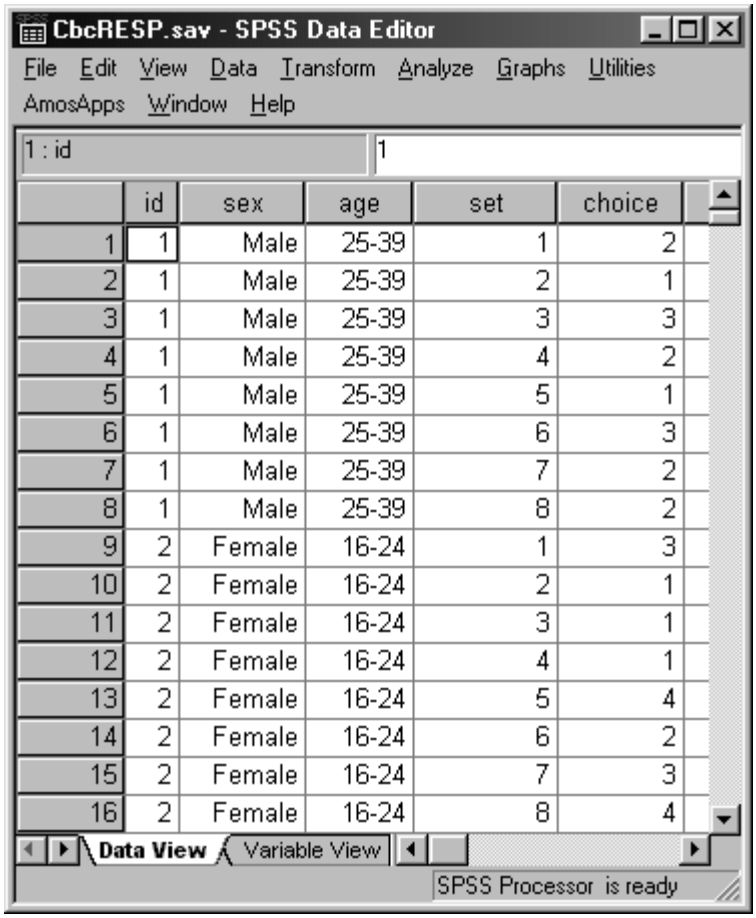
The segments differ on AGE and GENDER as follows:

Gender and Age distributions for each segment:

| | Segment | | |
|---|---|---|---|
| **GENDER** | 1 | 2 | 3 |
| Male | 0.25 | 0.40 | 0.83 |
| Female | 0.75 | 0.60 | 0.17 |
| | | | |
| **AGE** | | | |
| <25 | 0.69 | 0.28 | 0.19 |
| 25-39 | 0.14 | 0.15 | 0.43 |
| 40+ | 0.20 | 0.57 | 0.38 |

The choices obtained from 400 individuals, generated from this hypothetical Universe, are contained in the Response File cbcRESP.sav.

**Figure 16. First 2 cases in the Response File 'cbcRESP.sav'**



CbcRESP.sav - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities AmosApps Window Help

1 : id | 1

| | id | sex | age | set | choice |
|---|---|---|---|---|---|
| 1 | 1 | Male | 25-39 | 1 | 2 |
| 2 | 1 | Male | 25-39 | 2 | 1 |
| 3 | 1 | Male | 25-39 | 3 | 3 |
| 4 | 1 | Male | 25-39 | 4 | 2 |
| 5 | 1 | Male | 25-39 | 5 | 1 |
| 6 | 1 | Male | 25-39 | 6 | 3 |
| 7 | 1 | Male | 25-39 | 7 | 2 |
| 8 | 1 | Male | 25-39 | 8 | 2 |
| 9 | 2 | Female | 16-24 | 1 | 3 |
| 10 | 2 | Female | 16-24 | 2 | 1 |
| 11 | 2 | Female | 16-24 | 3 | 1 |
| 12 | 2 | Female | 16-24 | 4 | 1 |
| 13 | 2 | Female | 16-24 | 5 | 4 |
| 14 | 2 | Female | 16-24 | 6 | 2 |
| 15 | 2 | Female | 16-24 | 7 | 3 |
| 16 | 2 | Female | 16-24 | 8 | 4 |

Data View | Variable View

SPSS Processor is ready

## The Goal

We wish to assess the ability of the Latent GOLD Choice program to unmix the data correctly to reflect the simulated structural relationships. (Since the design for this choice experiment has relatively low efficiency, it is unclear whether it is possible to uncover the 3 underlying segments.) The BIC statistic will be used to determine which model fits best.

## Setting up the analysis

For this example, you can either open the setup file '1class.lgf' saved previously or go through the setup steps one at a time.

- To open the setup file, from the menus choose:

File

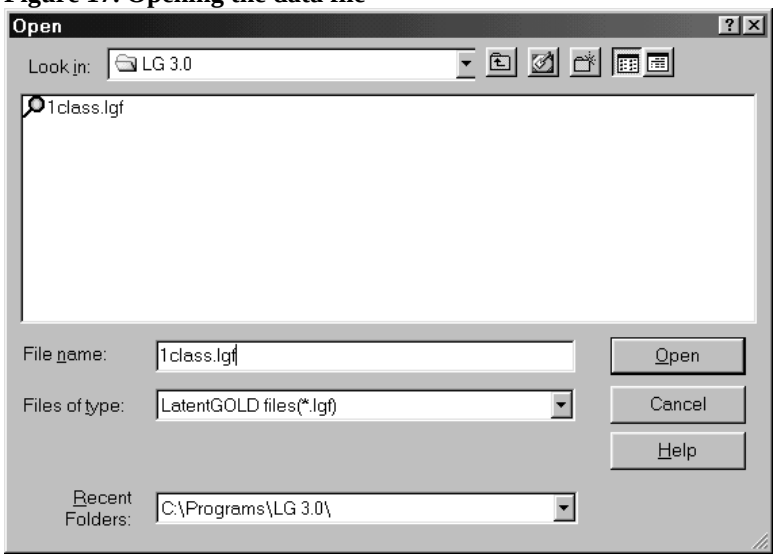
Open

- From the Files of type drop down list, select ‘LatentGOLD files (.lgf)’ if this is not already the default listing.

All files with the .lgf extensions appear in the list (see Figure below)

Note: If you copied the sample data file to a directory other than the default directory, change to that directory prior to retrieving the file.

**Figure 17. Opening the data file**




- Select ‘1class.lgf’ and click Open.
- Double-click on Model1 to bring up the 1-class model
- In the Classes box, type ‘1-4’ in place of 1 to request estimation of 4 models between 1 and 4 classes
- Skip ahead to *Estimating Models* (p. 11)

Alternatively, to go through the setup steps one at a time,

- from the menus choose:

File

Open

- From the Files of type drop down list, select SPSS system files (.sav) if this is not already the default listing.

All files with the .sav extensions appear in the list.

- Select ‘cbcRESP.sav’ and click Open

- Right click on ‘Model1’ and select Choice

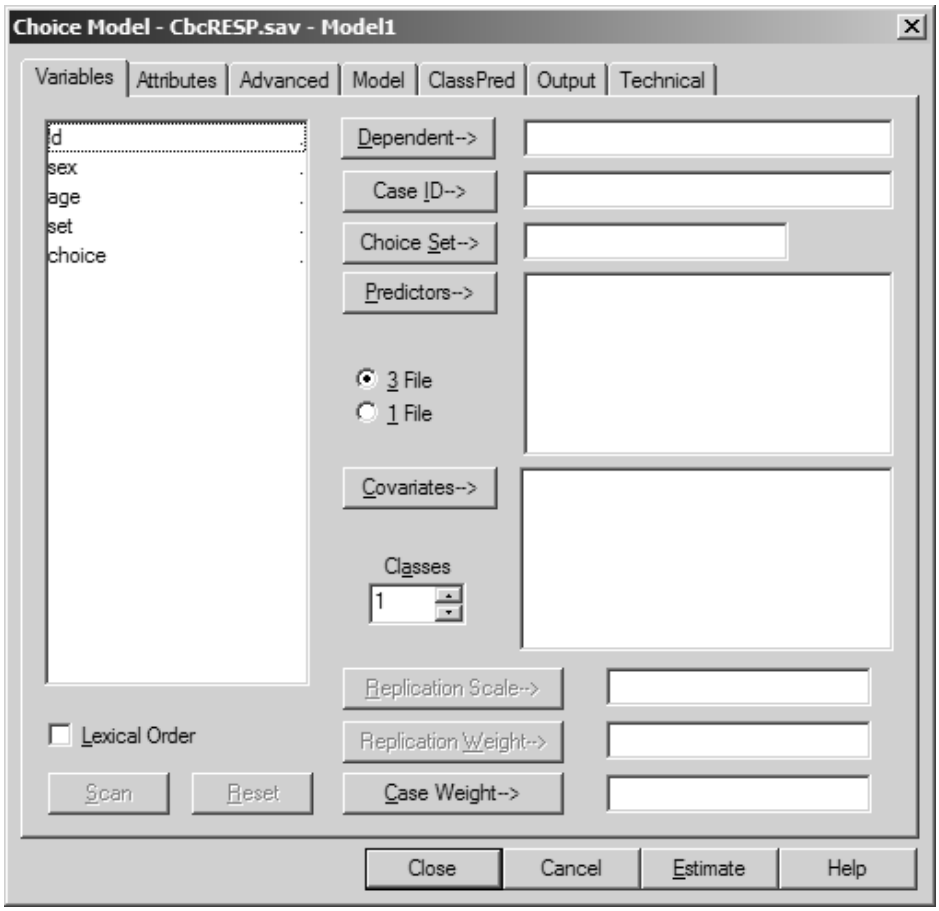



**Figure 18. Model Analysis Dialog Box**

- Select each variable and move it to the appropriate box by clicking the buttons to the left of these boxes.
  - ID to Case ID
  - SEX, AGE to Covariates
  - SET to Choice Set
  - CHOICE to Dependent
- In the Classes box, type '1-4' in place of 1 to request estimation of 4 models between 1 and 4 classes
- Right click on SEX and AGE and select 'Nominal' to change the scale type for these covariates.

Your Choice model setup should now look like this:

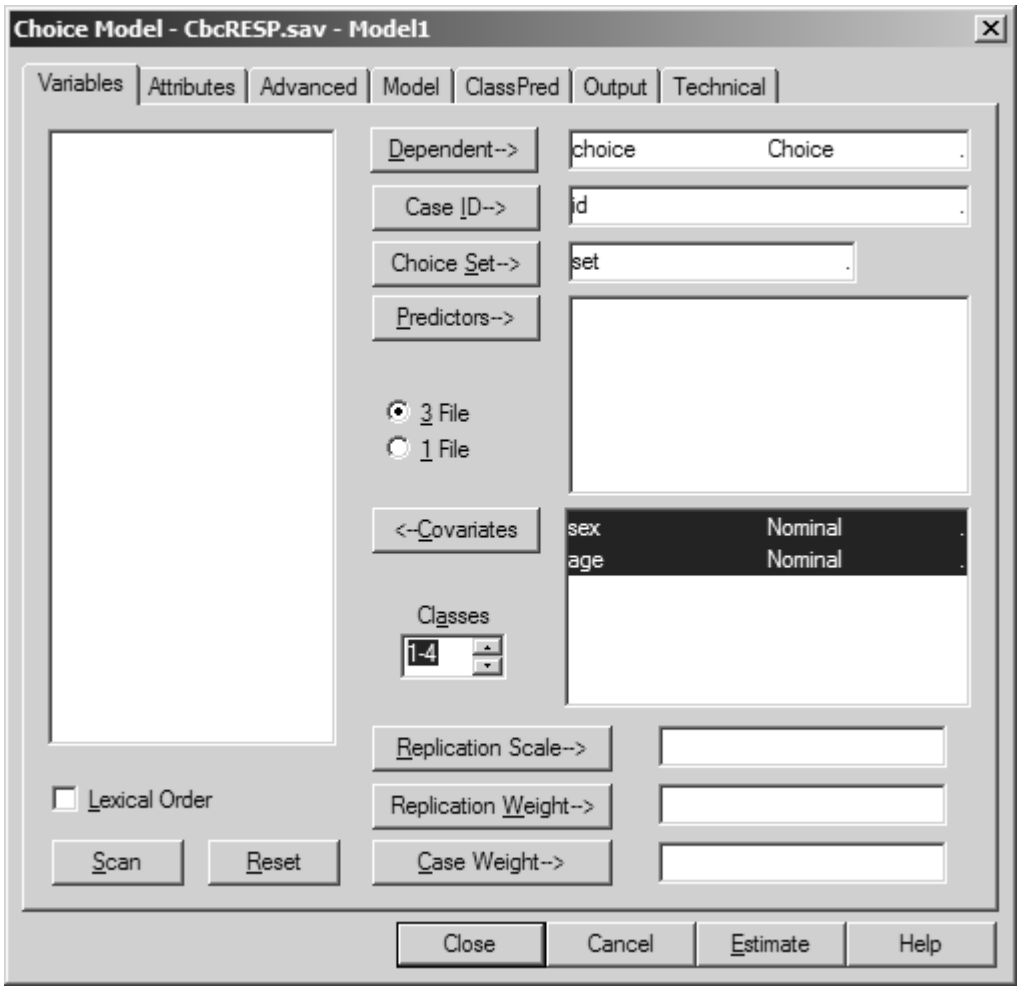



**Figure 19. Model Analysis Dialog Box after setup**

To complete the setup, we need to connect the Alternatives and Sets files to the Response File, and specify those attributes for which utilities are to be estimated.

- Open the Attributes tab by clicking on 'Attributes' at the top of the setup screen:

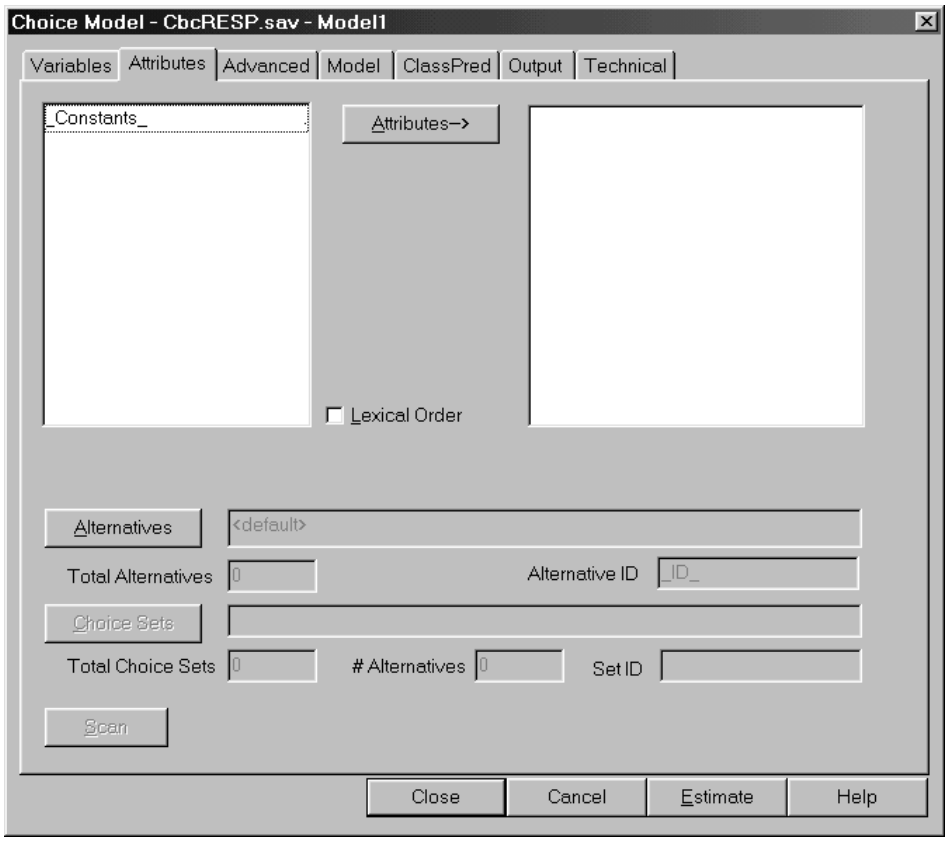



**Figure 20: Attributes Tab**

- Click the Alternatives button to display a list of files
- Select 'cbcALT11.sav' and click Open

In response to the prompt to select an ID variable,

- Select 'PRODCODE' and click OK

**Figure 21. Select ID Variable prompt**

The attribute variables from this file are now included in the variable list along with the alternative-specific constants variable _Constants_ , which is generated automatically by the program.

- Click the Choice Sets button to display a list of files
- Select 'cbcSET.sav' and click Open

In response to the prompt to select an ID variable,
- Select 'setid' and click OK

## Selecting the Variables for the Analysis

For this analysis, we will estimate main effects for all 4 attributes (FASHION, QUALITY, PRICE, and NONE) and a quadratic price effect by including the variable PRICESQ. To select these variables:

- Select all variables except for '_Constants_' and click Attributes to move them to the Attributes list box (or double click each variable that you want to move to the Attributes box and then click *Attributes*).

The designated variables now appear in the Attributes list box.

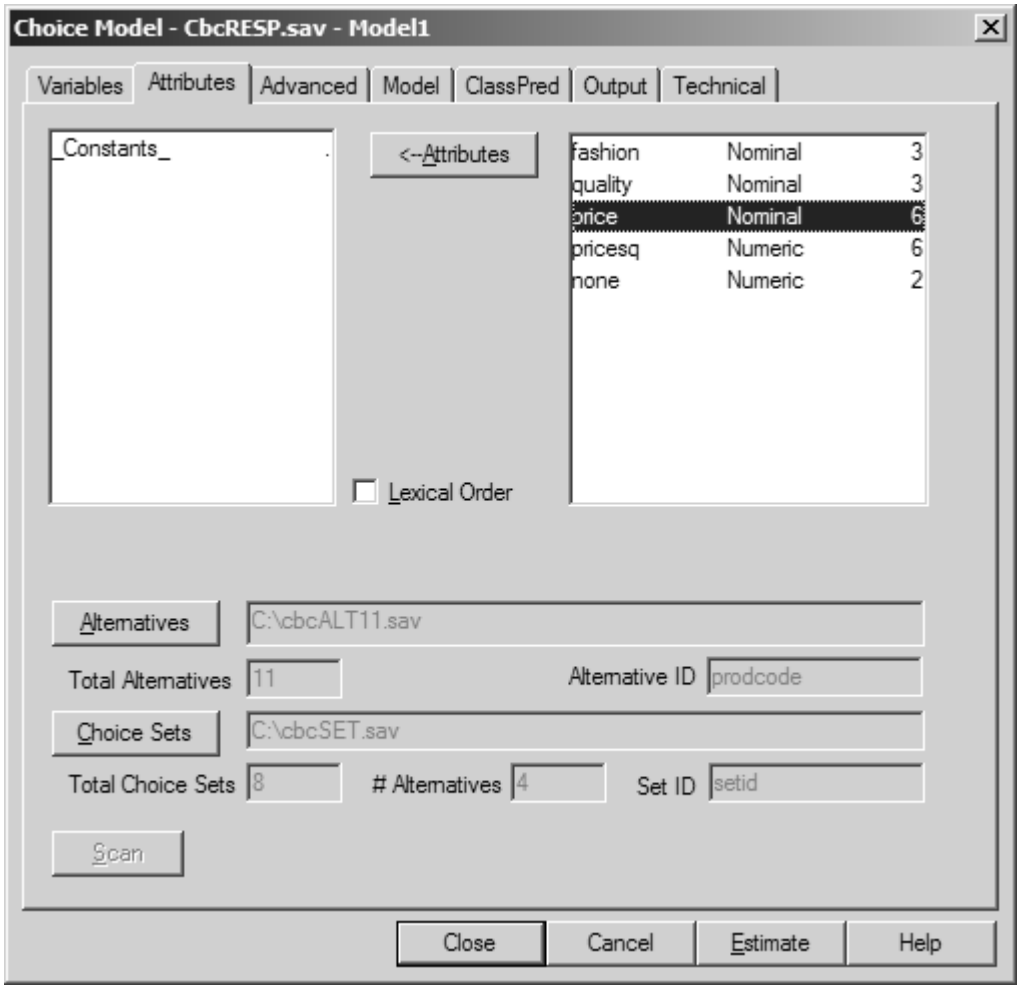


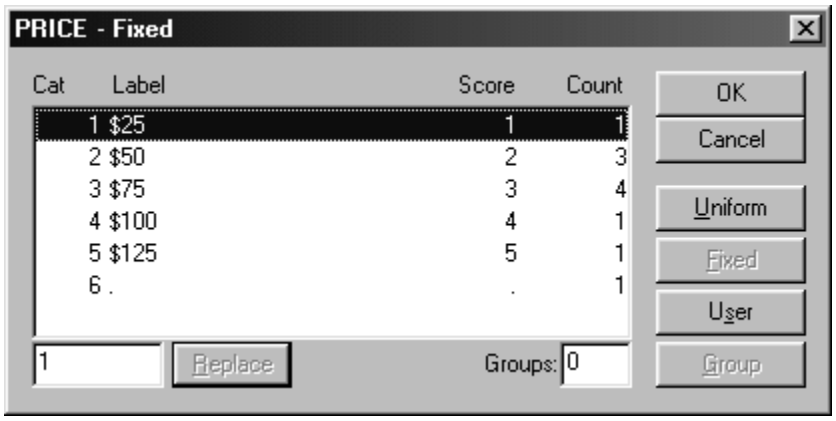

**Figure 22. Selecting the Attributes**

By default, character variables and variables containing consecutive integers (such as FASHION, QUALITY and PRICE) are treated as Nominal.

To view the coding for any variable, double-click on that variable.

- Double click on PRICE to view the consecutive integer scores assigned to the price levels.
- Click Cancel to close this Score window

**Figure 23. PRICE Score window**




To change the scale type of PRICE to numeric,

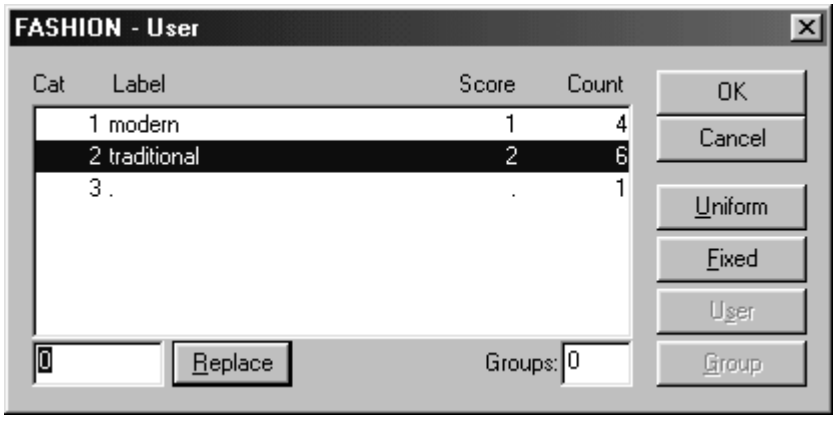
- Right click on PRICE, and select 'Numeric'

To change from effects coding (the default for Nominal variables) to '0,1' dummy coding for FASHION and QUALITY we can change the scale type to Numeric and change the scores to 0,1.

- Select FASHION and QUALITY
- Right click and select Numeric

- Double click on FASHION to retrieve the score box
- Double click on the second category
- Replace '2' with '0' in the Replace box
- Click Replace
- Click OK

**Figure 24. FASHION Score window**



Repeat he process for QUALITY

- Double click on QUALITY to retrieve the score box
- Double click on the second category
- Replace '2' with '0' in the Replace box
- Click Replace
- Click OK

Alternatively, dummy coding could be accomplished by maintaining the Nominal scale types for these 2 attributes, and selecting 'Dummy Last' in the 'Coding Nominal' section of the Output tab. Both approaches change to dummy coding, the only difference being in the display of the effect estimates in the Parameters Output. Only a single effect estimate is shown for each attribute using the former method, while the latter associates this effect estimate with the 1st category and displays a '0' effect for the 2nd category.

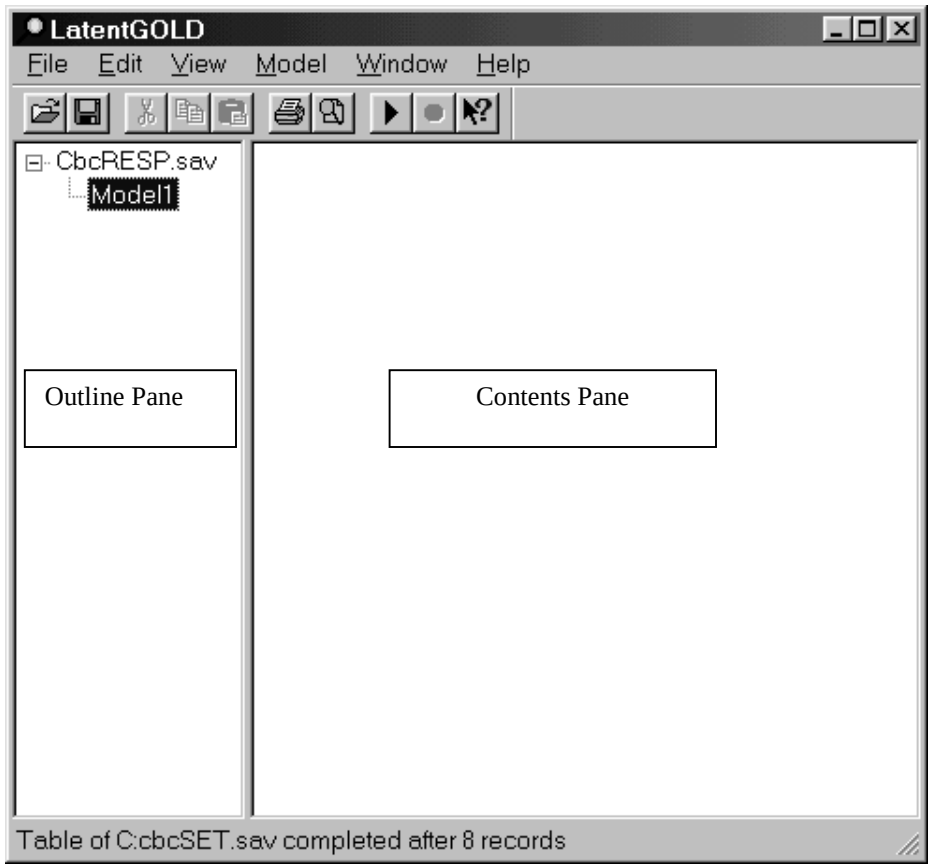
## Estimating Models

Now that we have specified the models, we are ready to estimate these models.

- Click Estimate (located at the bottom right of the analysis dialog box).

The setup window now closes and estimation begins. When Latent GOLD Choice completes the estimation, the model $L^2$, which assesses how well the model fits the data, and a list of various output files appears in the Outline pane.

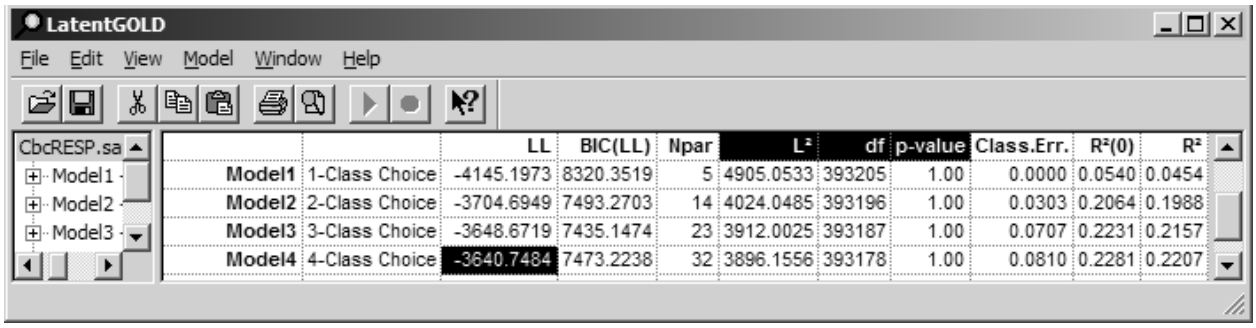
**Figure 25. The two Panes**



The Outline pane contains the name of the data file and a list of any previously estimated models (and their output) previously estimated. The Contents pane (currently empty) is where you will view the various types of output.

## Viewing Output and Interpreting Results

- Highlight the data file name 'cbcRESP.sav' and a summary of all the models estimated on this file appears in the Contents pane.




| | | LL | BIC(LL) | Npar | L² | df | p-value | Class.Err. | R²(0) | R² |
|---|---|---|---|---|---|---|---|---|---|---|
| **Model1** | 1-Class Choice | -4145.1973 | 8320.3519 | 5 | 4905.0533 | 393205 | 1.00 | 0.0000 | 0.0540 | 0.0454 |
| **Model2** | 2-Class Choice | -3704.6949 | 7493.2703 | 14 | 4024.0485 | 393196 | 1.00 | 0.0303 | 0.2064 | 0.1988 |
| **Model3** | 3-Class Choice | -3648.6719 | 7435.1474 | 23 | 3912.0025 | 393187 | 1.00 | 0.0707 | 0.2231 | 0.2157 |
| **Model4** | 4-Class Choice | -3640.7484 | 7473.2238 | 32 | 3896.1556 | 393178 | 1.00 | 0.0810 | 0.2281 | 0.2207 |

**Figure 26. Model Summary Output**

Note: For the 4-class model, you may obtain a local solution (LL = -3644.3868) rather than the global solution reported above (LL = -3640.7484). If this occurs, re-restimate the 4-class model.

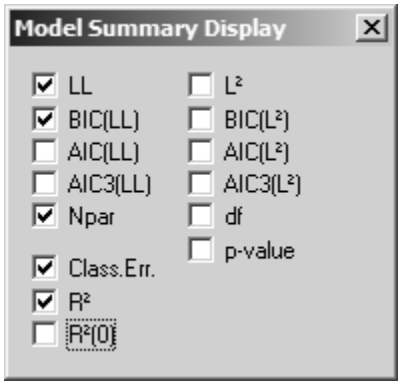
For repeated measure applications such as this, data sparseness exists which implies that the $L^2$ statistic does not follow a chi-square distribution. In such cases the reported p-value is not correct and the number of degrees of freedom is not informative. Hence, we will exclude the $L^2$, df and associated p-value columns from our Model Summary output.

In addition, notice that 2 different $R^2$ measures are displayed. For this model, we use the standard $R^2$. (For other applications as in tutorial 3, it will be more appropriate to utilize $R^2(0)$ rather than $R^2$ to assess the goodness of model prediction. The baseline for $R^2(0)$ is a null model containing no predictors at all, not even the variable '*_Constants_*'. This latter null model predicts each alternative to be equally likely to be selected, while the former null model predicts each alternative to be selected with probability equal to the overall observed marginal distribution.)

To remove the undesired columns from the Model Summary:

- Right click in the Contents Pane to retrieve the Model Summary Display
- Click in the boxes to the left of the $L^2$, df and p-value to remove the checkmarks from these items
- Close the Model Summary Display (by pressing Escape or clicking the X in the upper right of the Display)

**Figure 27. Model Summary Display**




The Model Summary in the Contents Pane now looks like this:

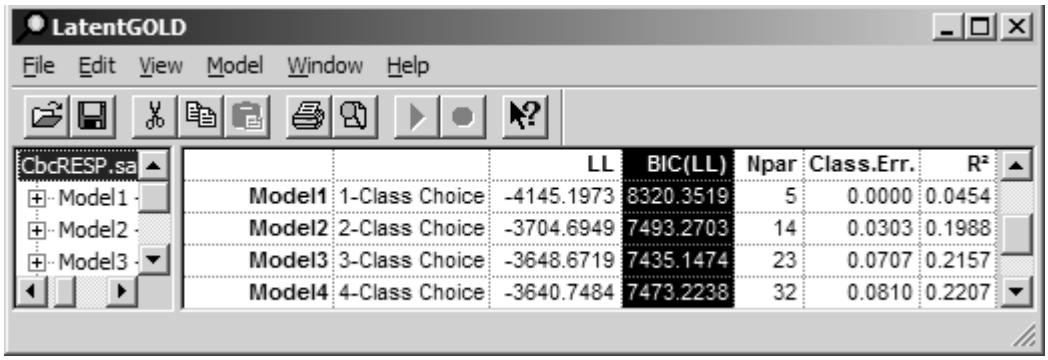

LatentGOLD
File Edit View Model Window Help
CbcRESP.sa
Model1
Model2
Model3
LL BIC(LL) Npar Class.Err. R²
Model1 1-Class Choice -4145.1973 8320.3519 5 0.0000 0.0454
Model2 2-Class Choice -3704.6949 7493.2703 14 0.0303 0.1988
Model3 3-Class Choice -3648.6719 7435.1474 23 0.0707 0.2157
Model4 4-Class Choice -3640.7484 7473.2238 32 0.0810 0.2207

**Figure 28. Updated Model Summary Display**

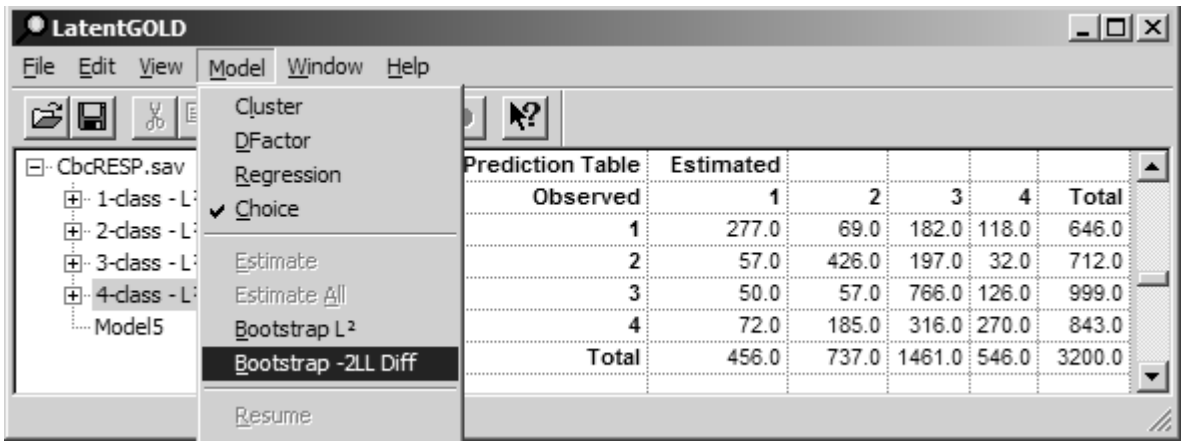
Notice that BIC criteria correctly identifies the 3-class model as best (lowest BIC). Notice also that the $R^2$ increases from .0454 for the standard aggregate (1-class) model to .2157 for the 3-class model. This $R^2$ statistic assesses the percentage variance explained in the dependent variable. In this example, the (nominal) dependent variable consists of 4 alternatives in each set (alt1, alt2, alt3 and alt4), variance is defined in terms of qualitative variance (Magidson, 1981), and the baseline or null model is the model that includes the alternative-specific constants '*_Constants_*' as the only predictor.

- In the Outline Pane, click once on 'Model 3' to select it and click again to enter Edit mode and rename it "3-class" for easier identification.
- Click once on each of the other models and rename each of them as well.

As a formal assessment of whether the $R^2$ obtained from the 4-class model provides a significant increase over the 3-class model, we can estimate a p-value for this improvement using the Conditional Bootstrap. By default, the Bootstrap procedure utilizes 500 samples and thus may take a few minutes to complete on your computer. Feel free to skip this, or if it is taking too long you may Cancel the estimation procedure by clicking the Red Stop button to Pause the model and selecting 'Abandon'.

- Select the model '4-class' in the Outline Pane
- Select 'Bootstrap –2LL Diff' from the Model menu

**Figure 29. Selecting Bootstrap –2LL Diff**




A list of models appears that contain eligible nested base models.

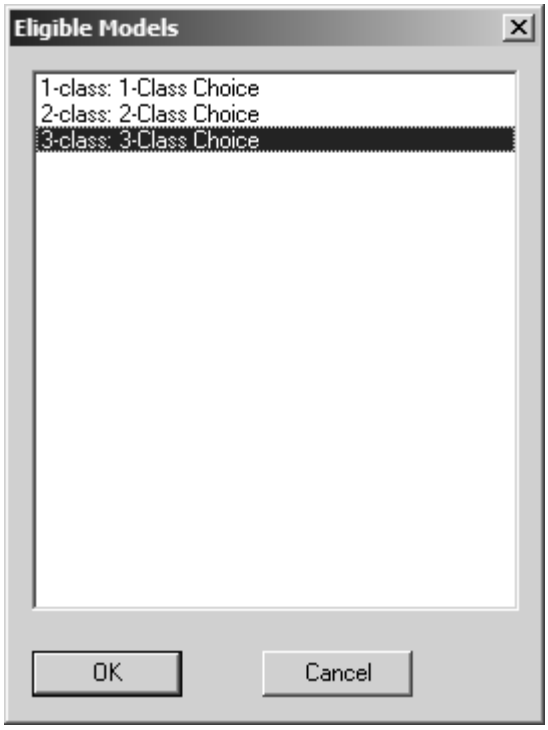



**Figure 30. List of Eligible Models**

- Select '3-class' from this list
- Select OK

When the procedure has completed, 2 additional model names appear in the Outline Pane – '3-classBoot' and '4classBoot'.

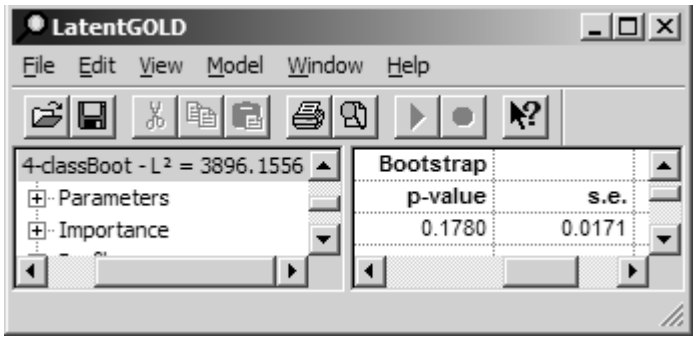



**Figure 31. 4-classBoot Model Summary**

Since the estimated p-value is larger than .05 (p=.178 with an estimated standard error of .-17 reported in the above Figure), the improvement by going to a 4th class is not statistically significant. The estimated p-value that you obtain should be close to this estimate.

## *Examining the Model Output files*

- Click on the expand icon (+) next to the 1-class model and several output files appear.
- Click on Parameters to view the part-worth utility estimates in the Contents Pane

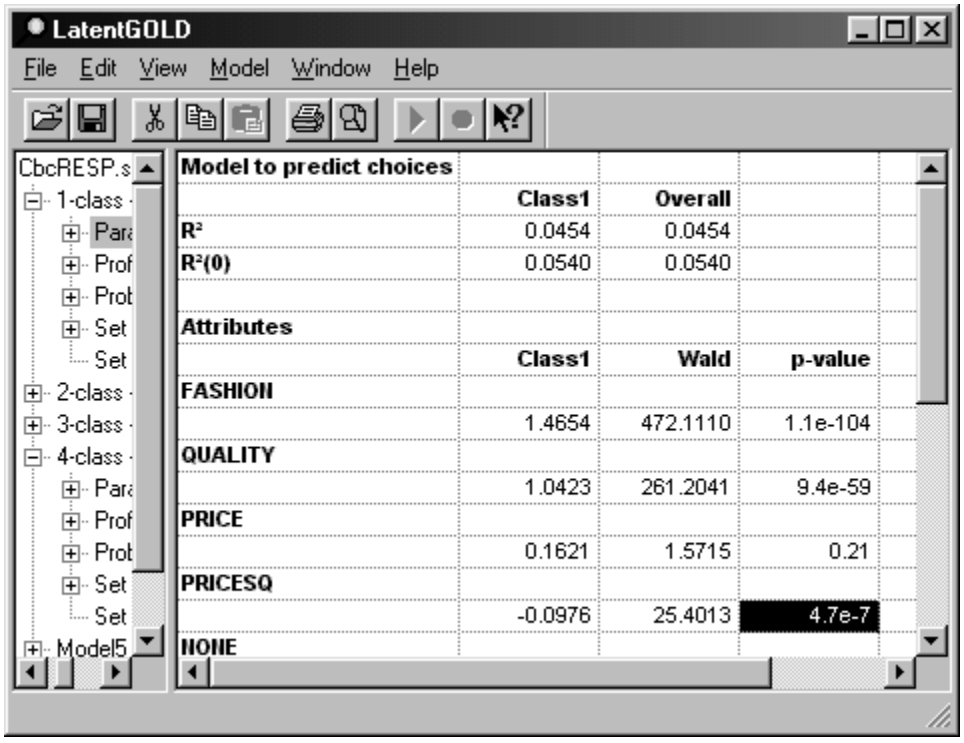
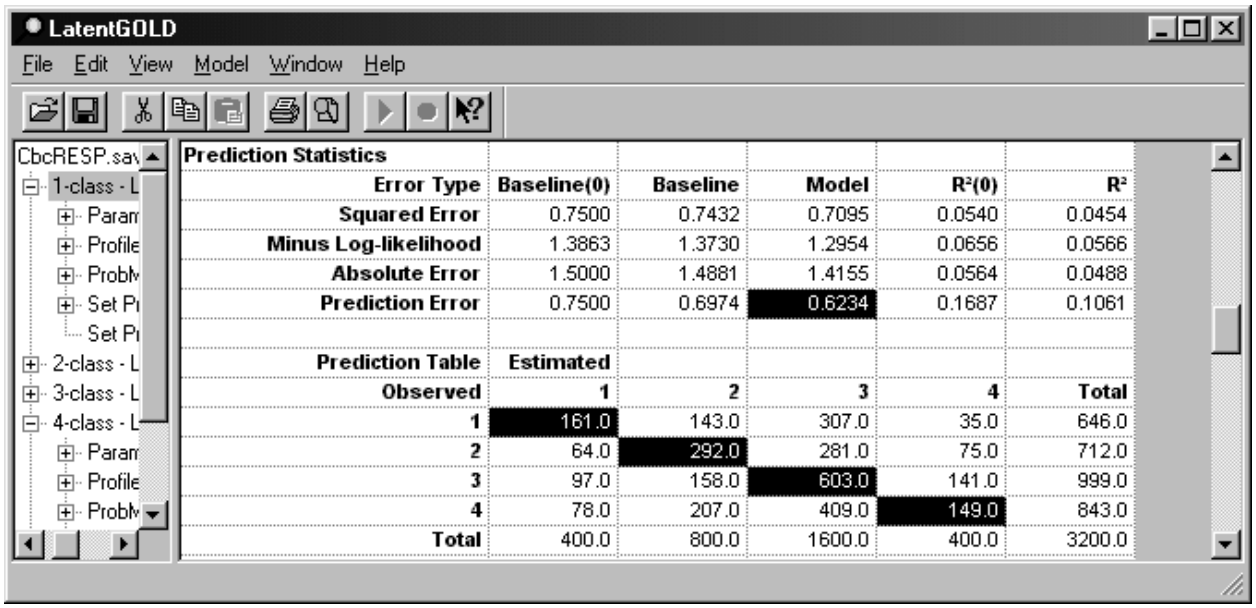



**Figure 32. Parameters Output for 1-class Model**

Notice that the effect of PRICESQ is highly significant ($p = 4.7 \times 10^{-7}$) in this model. Since the effect of PRICESQ is 0 in the true model, this example shows that the effects (and predictions) obtained from the aggregate model can not be trusted. When choices are based on different utilities (in different population segments), estimates obtained under the aggregate model will typically be biased.

- Click on '1-class' and scroll down to the Prediction Statistics section

**Figure 33. Prediction Statistics for 1-class Model**




**Prediction Statistics**

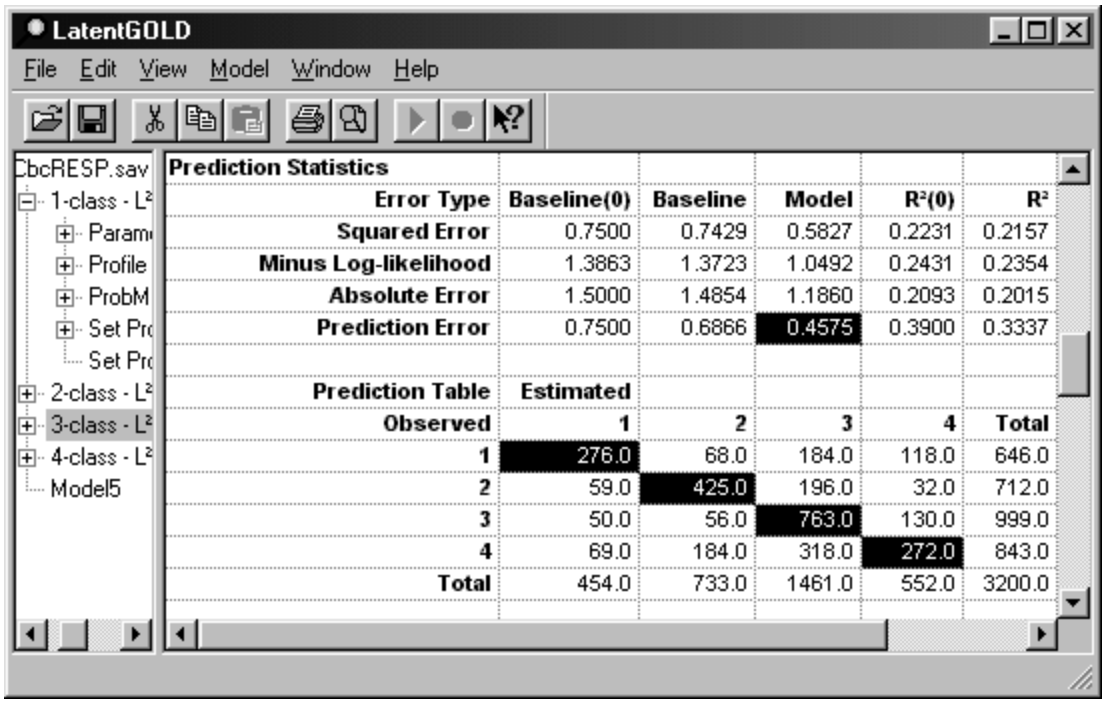
| Error Type | Baseline(0) | Baseline | Model | $R^2(0)$ | $R^2$ |
|---|---|---|---|---|---|
| Squared Error | 0.7500 | 0.7432 | 0.7095 | 0.0540 | 0.0454 |
| Minus Log-likelihood | 1.3863 | 1.3730 | 1.2954 | 0.0656 | 0.0566 |
| Absolute Error | 1.5000 | 1.4881 | 1.4155 | 0.0564 | 0.0488 |
| Prediction Error | 0.7500 | 0.6974 | 0.6234 | 0.1687 | 0.1061 |

| Prediction Table | Estimated | | | | |
|---|---|---|---|---|---|
| Observed | 1 | 2 | 3 | 4 | Total |
| 1 | 161.0 | 143.0 | 307.0 | 35.0 | 646.0 |
| 2 | 64.0 | 292.0 | 281.0 | 75.0 | 712.0 |
| 3 | 97.0 | 158.0 | 603.0 | 141.0 | 999.0 |
| 4 | 78.0 | 207.0 | 409.0 | 149.0 | 843.0 |
| Total | 400.0 | 800.0 | 1600.0 | 400.0 | 3200.0 |

The prediction table shows that this 1-class (aggregate) model correctly predicts only 161 of the 646 alt1 responses, only 292 of the 712 alt2 responses, 603 of the 999 alt3 responses and only 149 of the 843 alt4 responses. Overall, only 1,205 (161 + 292 + 603 +149) of the total 3,200 observed choices are predicted correctly – a 'hit rate' of only 37.66%. This represents a Prediction Error of 1-.3766 = .6234 (as reported in the row of the output labeled 'Prediction Error').

For comparison, we will look at the corresponding statistics under the 3-class model.

- Click on '3-class' and scroll down to the Prediction Statistics section




Prediction Statistics

| Error Type | Baseline(0) | Baseline | Model | $R^2(0)$ | $R^2$ |
|---|---|---|---|---|---|
| Squared Error | 0.7500 | 0.7429 | 0.5827 | 0.2231 | 0.2157 |
| Minus Log-likelihood | 1.3863 | 1.3723 | 1.0492 | 0.2431 | 0.2354 |
| Absolute Error | 1.5000 | 1.4854 | 1.1860 | 0.2093 | 0.2015 |
| Prediction Error | 0.7500 | 0.6866 | 0.4575 | 0.3900 | 0.3337 |

| Prediction Table | Estimated | | | | |
|---|---|---|---|---|---|
| Observed | 1 | 2 | 3 | 4 | Total |
| 1 | 276.0 | 68.0 | 184.0 | 118.0 | 646.0 |
| 2 | 59.0 | 425.0 | 196.0 | 32.0 | 712.0 |
| 3 | 50.0 | 56.0 | 763.0 | 130.0 | 999.0 |
| 4 | 69.0 | 184.0 | 318.0 | 272.0 | 843.0 |
| Total | 454.0 | 733.0 | 1461.0 | 552.0 | 3200.0 |

**Figure 34. Prediction Statistics for 3-class Model**

By accounting for the heterogeneity among the 3 segments, the prediction error has been reduced to .4575 by the 3-class model.

- Click on the expand icon (+) next to the 3-class model and select Parameters.

The part-worth utility estimates appear for each class.

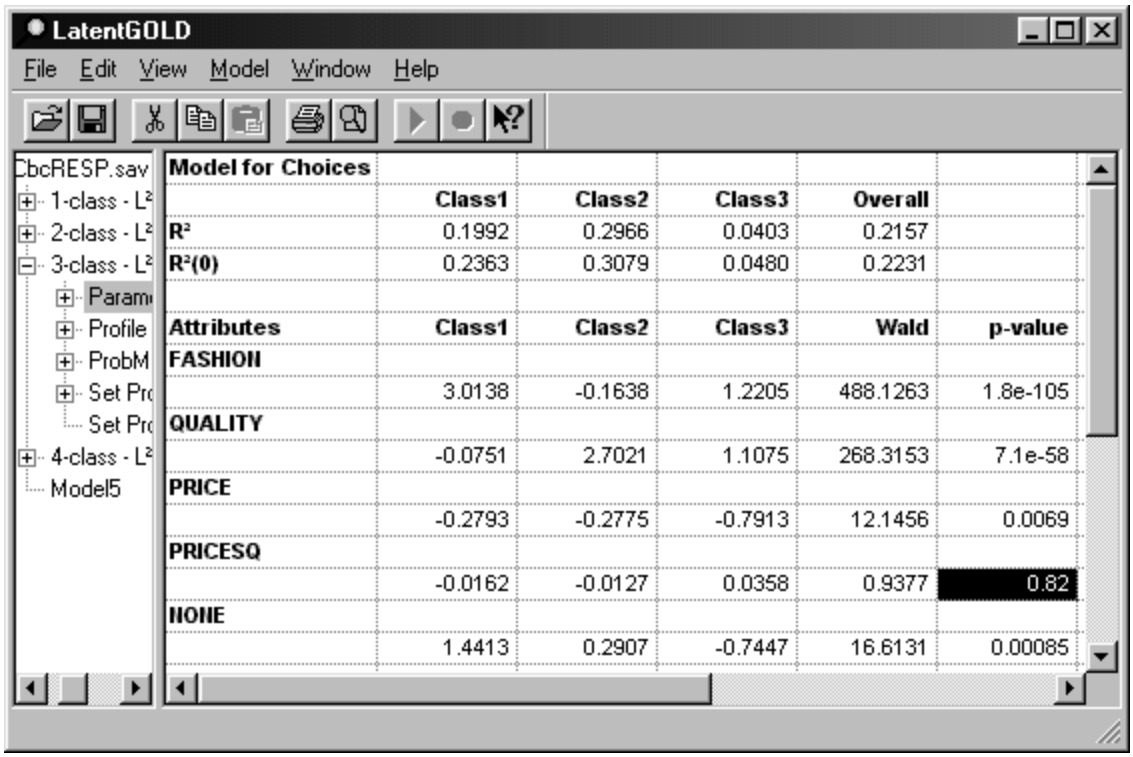



**Figure 35. Parameters Output for 3-class Model**

## *Using Restrictions to Refine the 3-Class Model*

Under the 3-class model, the estimates for PRICESQ are no longer significant, which reflects the true structure of this model. The associated p-value is greater than 0.05 (p = .82).

- Double click on '3-class' to re-open the model setup window.
- Click Attributes to open the attributes tab
- Select PRICESQ and click the Attributes button to remove this effect from the model.
- Click Estimate to re-estimate the model.
- After the estimation has completed click Parameters to view the new parameters estimates.

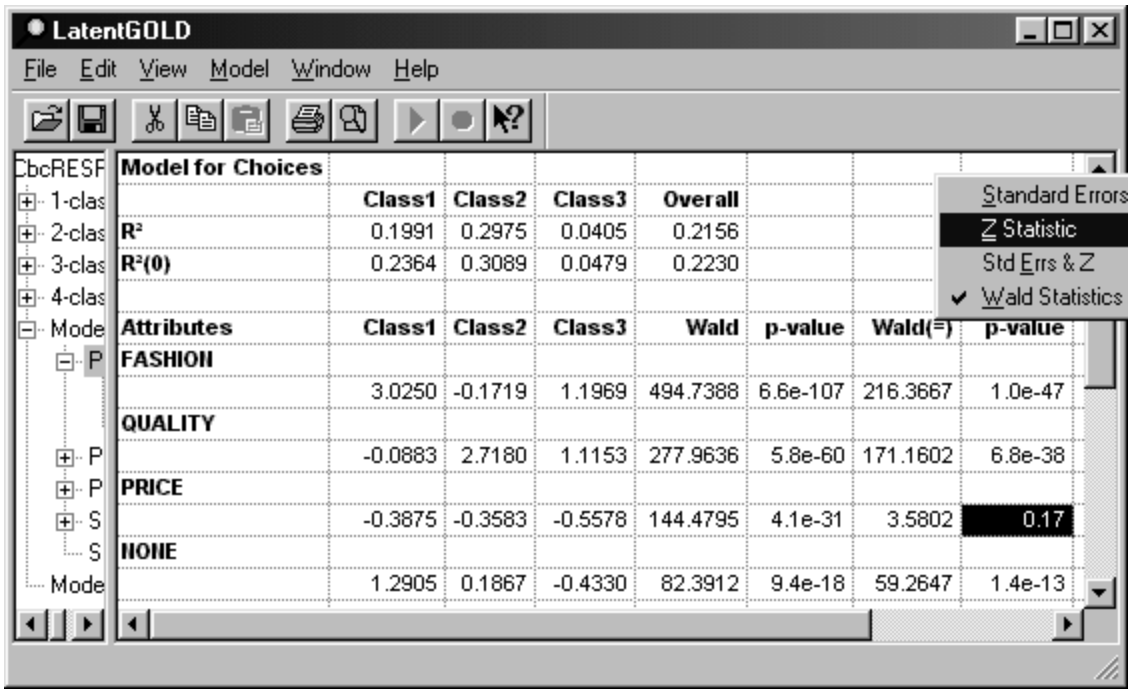



**Figure 36. New Parameters Output**

Notice that the within-class estimates for PRICE are close to each other. The formal test of equality is given by the Wald(=) statistic, for which the p-value (0.17) is not significant. Thus, we will restrict the effect of PRICE to be equal across all 3 segments.

To view whether any of the parameter estimates are non-significant:

- Right click on the Contents Pane and select Z Statistic from the Popup menu.

The Z-statistics show that for class 1 the effect of higher quality, and for class 2 the effect of modern fashion are not significant.

To implement these parameter restrictions

- Double click on the last model ('Model5' here) to re-open the model setup screen.
- Click on 'Model' to Open the Model tab.
- Right click under column 1 in the QUALITY row and select 'No Effect' from the pop-up menu to restrict the class 1 effect for QUALITY to zero.
- Right click under column 2 in the FASHION row and select 'No Effect' from the pop-up menu to restrict the class 1 effect for FASHION to zero.
- Right click under Class Independent column in the PRICE row and select 'Yes' to restrict the effect of PRICE to be class independent.

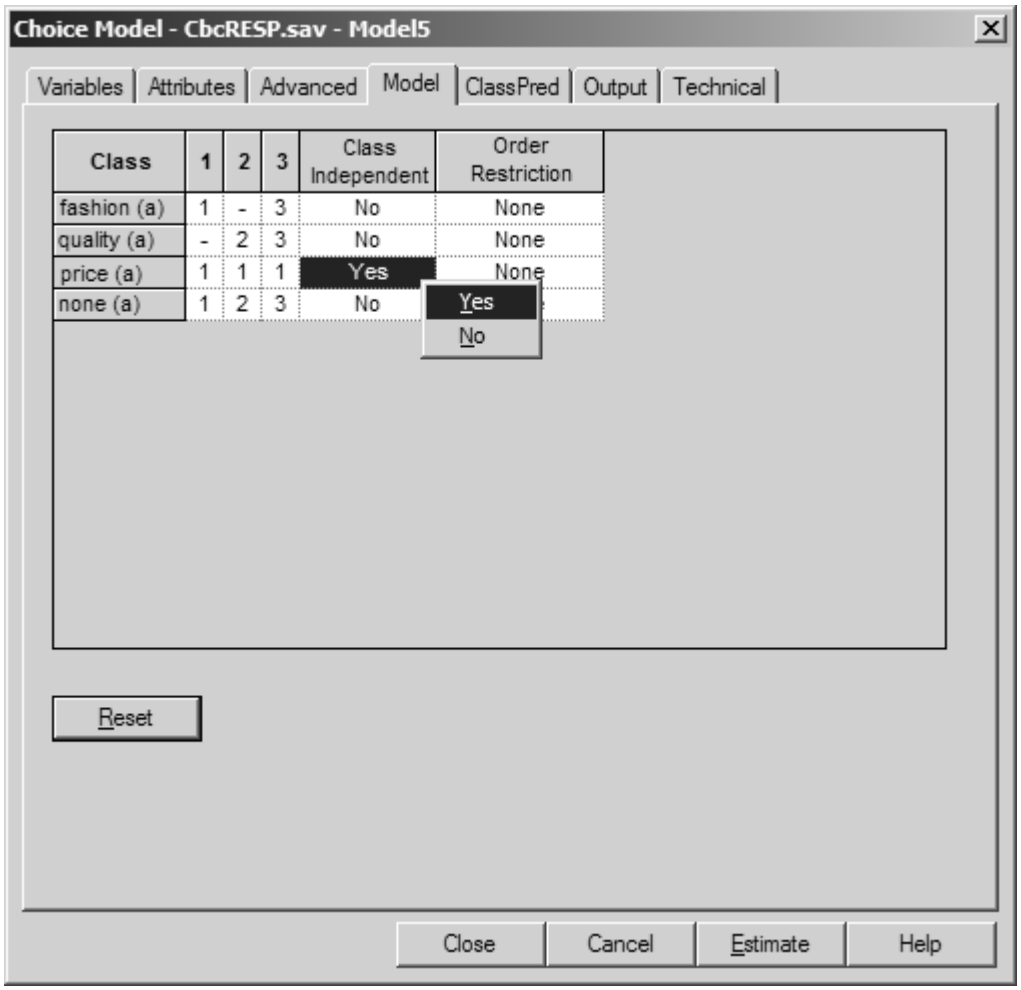
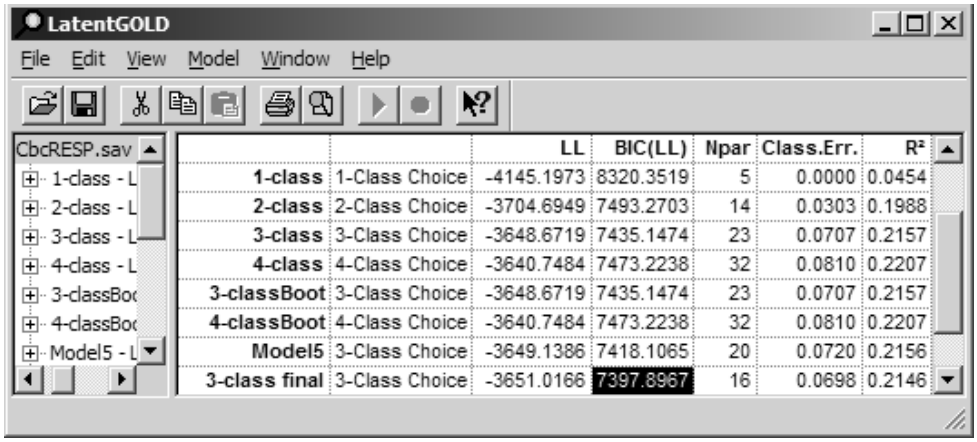



**Figure 37. Implementing Parameter Restrictions**

- Click Estimate to re-estimate the model.
- Re-name this model '3-Class Final'

Notice that the BIC for this final model is now the best.




| | | LL | BIC(LL) | Npar | Class.Err. | R² |
|---|---|---|---|---|---|---|
| **1-class** | 1-Class Choice | -4145.1973 | 8320.3519 | 5 | 0.0000 | 0.0454 |
| **2-class** | 2-Class Choice | -3704.6949 | 7493.2703 | 14 | 0.0303 | 0.1988 |
| **3-class** | 3-Class Choice | -3648.6719 | 7435.1474 | 23 | 0.0707 | 0.2157 |
| **4-class** | 4-Class Choice | -3640.7484 | 7473.2238 | 32 | 0.0810 | 0.2207 |
| **3-classBoot** | 3-Class Choice | -3648.6719 | 7435.1474 | 23 | 0.0707 | 0.2157 |
| **4-classBoot** | 4-Class Choice | -3640.7484 | 7473.2238 | 32 | 0.0810 | 0.2207 |
| **Model5** | 3-Class Choice | -3649.1386 | 7418.1065 | 20 | 0.0720 | 0.2156 |
| **3-class final** | 3-Class Choice | -3651.0166 | 7397.8967 | 16 | 0.0698 | 0.2146 |

**Figure 38. Updated Model Summary Output**

To remove the Bootstrap models,

- Select '3-classBoot'
- Select Delete from the Model menu
- Repeat this for '4-classBoot'

## *Profile Output: viewing Re-scaled Parameters*

- Click on the expand icon (+) next to 3-class final model to display the output files
- Click on Profile to display the Profile Output in the Contents Pane

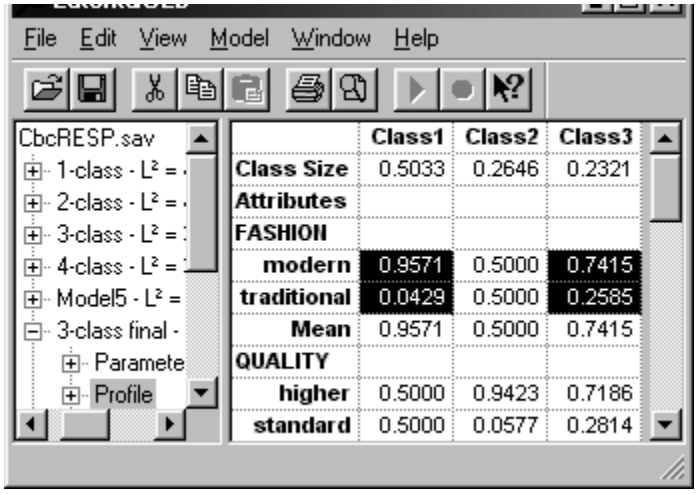


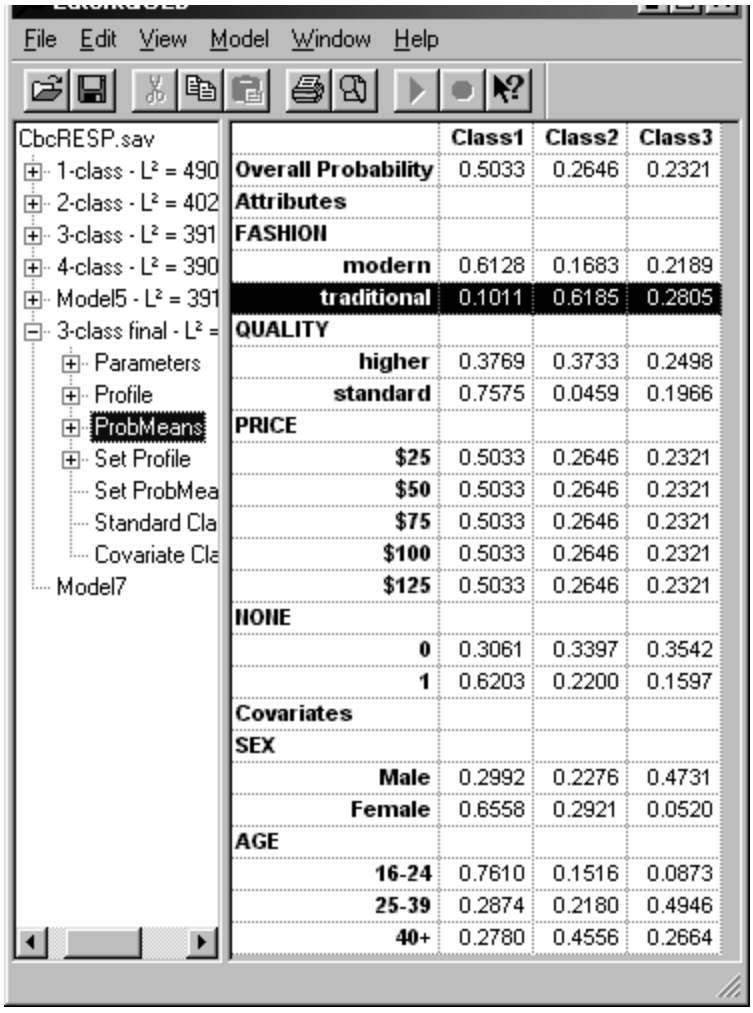

**Figure 39. Final 3-class Model Profile Output**

The Profile output consists of the segment size estimates together with re-scaled parameter estimates that correspond to column percentages. Notice that the segment size estimates of .50, .26 and .23 are very close to the true universe parameters.

These re-scaled parameters for FASHION and QUALITY have a nice probabilistic interpretation. The probability of choosing the Modern over the Traditional style when faced with just these 2 alternatives where the QUALITY and PRICE were identical is estimated to be .9571 for persons in segment 1, .5 for persons in segment 2, and .7415 for persons in segment 3. A similar interpretation is available for QUALITY. When faced with a choice between 1) a pair of Higher quality and 2) a pair of Standard quality shoes – both having the same price and same style (Modern or Traditional), the probability of choosing the Higher quality shoes is .5 for segment 1, .9423 for segment 2 and .7186 for segment 3. While the re-scaled parameters for PRICE can not be interpreted in same way, they still convey information about which segment(s) are more price sensitive.

A second benefit of the re-scaled parameters is that when presented as row percentages (as opposed to column percentages) they become the ProbMeans output, which yields an informative display in the dimension of the segments.

- Click on ProbMeans to display the corresponding row percentages.




| | Class1 | Class2 | Class3 |
|---|---|---|---|
| **Overall Probability** | 0.5033 | 0.2646 | 0.2321 |
| **Attributes** | | | |
| **FASHION** | | | |
| **modern** | 0.6128 | 0.1683 | 0.2189 |
| **traditional** | 0.1011 | 0.6185 | 0.2805 |
| **QUALITY** | | | |
| **higher** | 0.3769 | 0.3733 | 0.2498 |
| **standard** | 0.7575 | 0.0459 | 0.1966 |
| **PRICE** | | | |
| **$25** | 0.5033 | 0.2646 | 0.2321 |
| **$50** | 0.5033 | 0.2646 | 0.2321 |
| **$75** | 0.5033 | 0.2646 | 0.2321 |
| **$100** | 0.5033 | 0.2646 | 0.2321 |
| **$125** | 0.5033 | 0.2646 | 0.2321 |
| **NONE** | | | |
| **0** | 0.3061 | 0.3397 | 0.3542 |
| **1** | 0.6203 | 0.2200 | 0.1597 |
| **Covariates** | | | |
| **SEX** | | | |
| **Male** | 0.2992 | 0.2276 | 0.4731 |
| **Female** | 0.6558 | 0.2921 | 0.0520 |
| **AGE** | | | |
| **16-24** | 0.7610 | 0.1516 | 0.0873 |
| **25-39** | 0.2874 | 0.2180 | 0.4946 |
| **40+** | 0.2780 | 0.4556 | 0.2664 |

**Figure 40. Final 3-class Model ProbMeans Output**

The row percentages highlighted above state that persons choosing Traditional vs. Modern style shoes (where price and quality are the same), are most likely (posterior probability = .6185) to be in segment 2. These row percentages can be used to position each category in an informative 2-dimensional barycentric coordinate display. Choices can also be appended to this plot.

- Click on the expand icon (+) next to ProbMeans
- Click on Tri-Plot to display the Plot
- Right click in the Contents Pane to display the Plot Control Panel
- Click on PRICE to remove PRICE from the plot
- Click on the drop down set list and select set #4
- Click on the Choice Set symbol to highlight the appended choices on the plot

Each vertex of the tri-plot corresponds to a segment. From the plot it can be seen that Class 2 (corresponding to the lower right vertex) is the segment most likely to prefer Higher Quality and Traditional style shoes. This segment is also tends to be the oldest (higher proportion of persons 40+) of the segments.

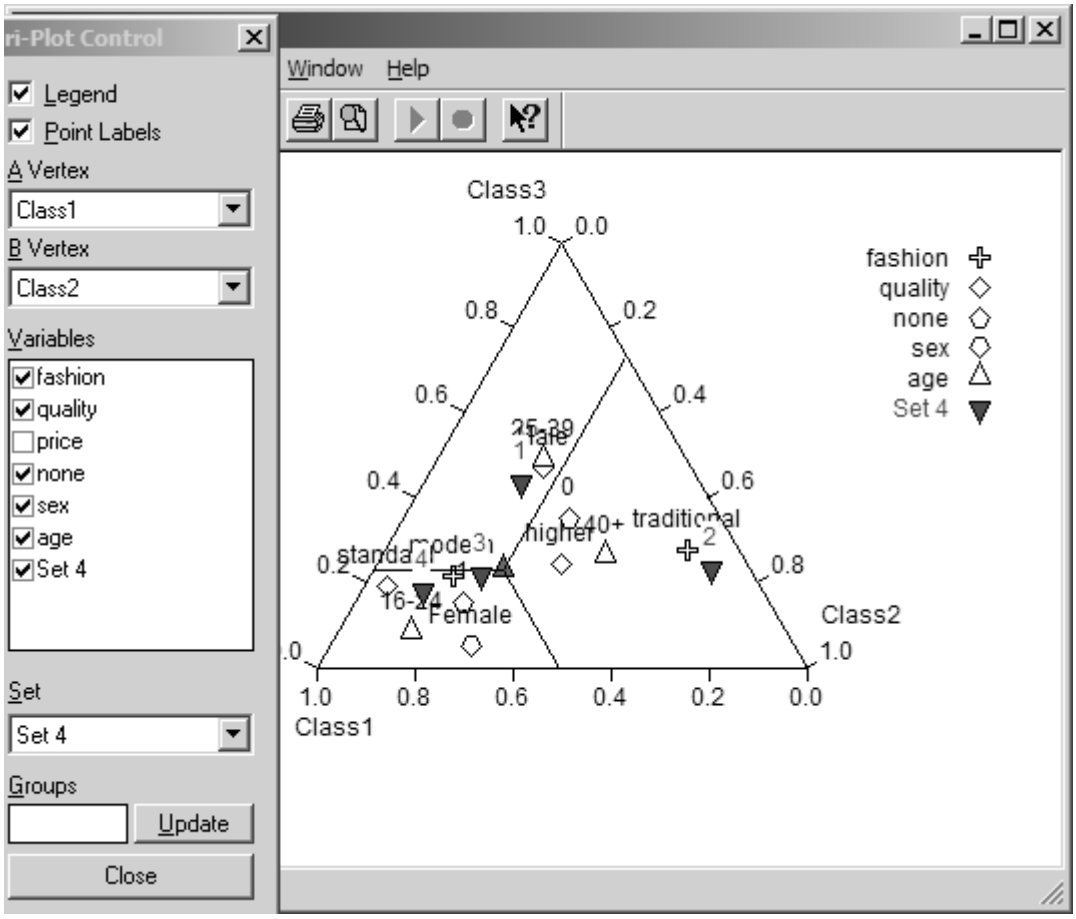



**Figure 41. Tri-Plot**

Set #4 consists of 1:TS2 2:TH4 3:MH5 4:None. It is clear from the plot that persons who choose alternative #2:TH4 from this set are highly likely to be in segment 2.

## *Additional Output Options*

- Latent GOLD Choice offers many additional output options.

Double click on '3-class final' to re-open the model setup screen

- Click on Output to Open the Output Tab

To obtain output to see how to classify each person into the most likely segment

- Click on Standard Classification
- Click on Covariate Classification

To append this information and predicted choices to the response file

- Click on ClassPred to Open the ClassPred Tab
- Click on Standard Classification
- Click on Covariate Classification
- Click on Predicted Values

To append individual HB-like coefficients to the response file

- Click on Individual Coefficients
- Replace the default output filename if desired

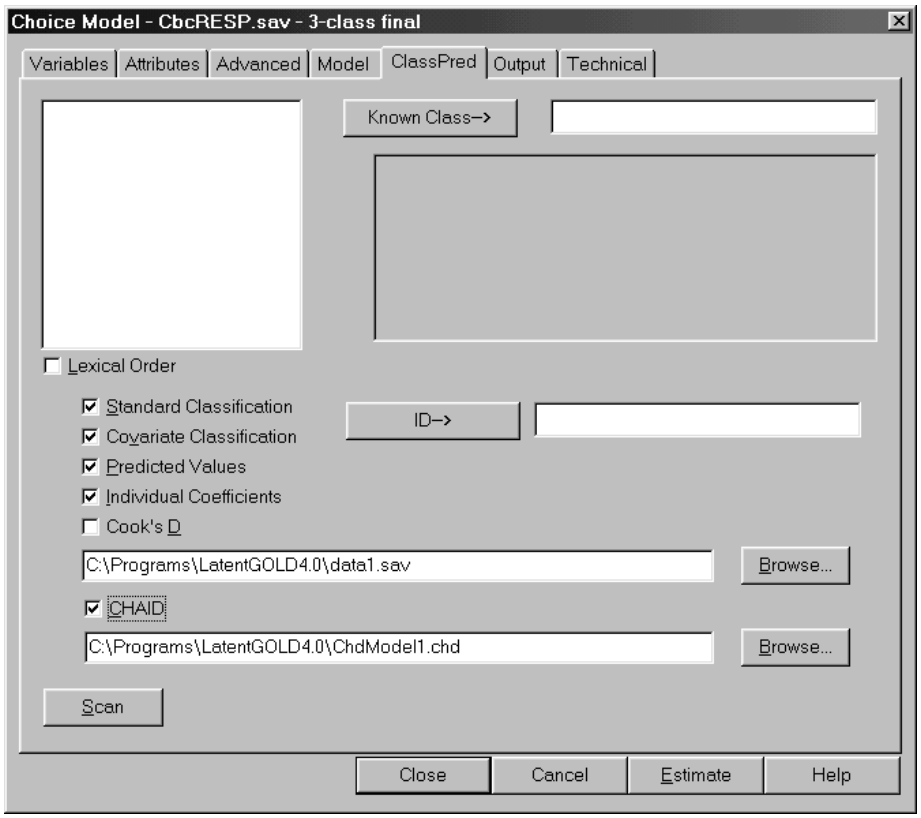



**Figure 42. ClassPred Tab**

Note that we also requested a CHAID input file to be created. We will illustrate this option later in tutorial 1A.

- Click Estimate

To open the standard classification output,

- Click Standard Classification in the Outline Pane

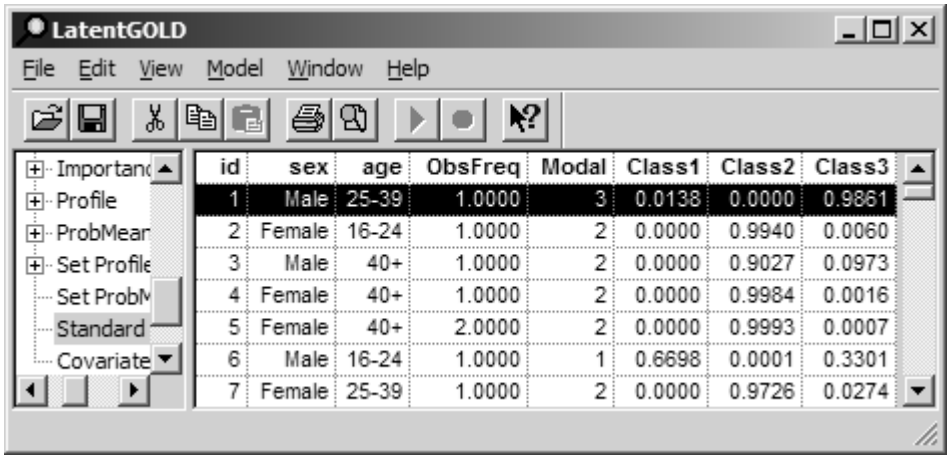



**Figure 43. Standard Classification Output**

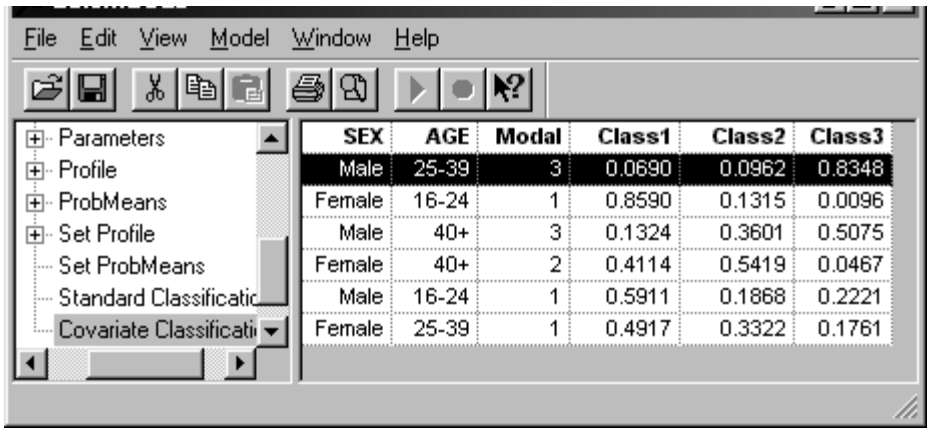
This output shows that case 1 is very likely (posterior probability = .9861) a member of segment 3.

The covariate classification information is useful for classifying new cases, which did not participate in the conjoint experiment.

- Click Covariate Classification

This table shows that based solely on this person's demographics, he would still be classified into segment 3 although with somewhat less certainty (probability = .8348). We will explore the relationship between the covariates and the classes more extensively in Tutorial 1A using the SI-CHAID 4.0 option.

**Figure 44. Covariate Classification Output**




Latent GOLD Choice was found to be successful in uncovering the 3-segment structure in this example. In tutorial #2 we will use our final model to generate choice predictions.

To save the analysis file for future work

- Click on the data file name cbcRESP.sav at the top of the outline pane
- From the File Menu, select Save Definitions
- Click Save